Linear Equations Containing Fractions

Sometimes it is simpler to solve an equation with fractions by first eliminating the fractions using a process called clearing fractions.

To clear fractions: clear parentheses first, then
multiply both sides of the equation by the LCD of all terms in the equation.

Determine which of the values could be used to clear fractions in the given equation.

1. $\frac{1}{6}x=\frac{2}{3}+\frac{x}{9}$

   Values: 3, 6, 12, 18, 24, 36

2. $\frac{3}{4}x+\frac{7}{8}=7-\frac{1}{2}x$

   Values: 2, 4, 7, 8, 12, 16

Solve the equation.

3. $\frac{1}{5}y+3=\frac{7}{10}$

4. $\frac{2}{3}x+7=\frac{5}{6}-x$

Method 1:

$\frac{1}{5}y+3=\frac{7}{10}$

$\frac{1}{5}y=\frac{7}{10}-3$

$\frac{1}{5}y=\frac{7}{10}-\frac{3}{1}\left(\frac{10}{10}\right)$

$\frac{1}{5}y=\frac{7}{10}-\frac{30}{10}$

$\frac{1}{5}y=-\frac{23}{10}$

$\left(\frac{5}{1}\right)\frac{1}{5}y=-\frac{23}{10}\left(\frac{5}{1}\right)$

$y=-\frac{23}{2}$

$\boxed{-\frac{23}{2}}$

#3 using method 2: multiply through to clear fractions

$\frac{1}{5}y+3=\frac{7}{10}$

multiply both sides by 10:

$10\left(\frac{1}{5}y+3\right)=\left(\frac{7}{10}\right)\frac{10}{1}$

$10\left(\frac{1}{5}y\right)+10(3)=\frac{7}{1}$

$\frac{10}{5}y+30=7$

$2y+30=7$

$-30 \quad -30$

$2y=-23$

$\frac{2y}{2}=\frac{-23}{2}$

$y=-\frac{23}{2}$

$\left\{-\frac{23}{2}\right\}$

4. $\frac{2}{3}x + 7 = \frac{5}{6} - x$

Solve by multiplying through to clear the fractions:

Multiply both sides by 6:

$$\frac{6}{1}\left(\frac{2}{3}x + 7\right) = \frac{6}{1}\left(\frac{5}{6} - x\right)$$

$$6\left(\frac{2}{3}x\right) + 6(7) = 6\left(\frac{5}{6}\right) - 6x$$

$$\frac{4}{1}x + 42 = \frac{5}{1} - 6x$$

$$4x + 42 = 5 - 6x$$

$$4x + 6x = 5 - 42$$

$$10x = -37$$

$$\frac{10x}{10} = \frac{-37}{10}$$

$$x = -\frac{37}{10}$$

$$\left\{-\frac{37}{10}\right\}$$

Ex: $\frac{3}{4}x - \frac{2}{3} + \frac{1}{6} = 4x - \frac{5}{2}$

multiply both sides (all terms) by 12:

$$\left(\frac{12}{1}\right)\frac{3}{4}x - \frac{2}{3}(12) + \frac{1}{6}(12) = 4x(12) - \frac{5}{2}(12)$$

$$\frac{36}{4}x - \frac{24}{3} + \frac{12}{6} = 48x - \frac{60}{2}$$

$$9x - 8 + 2 = 48x - 30$$

(depends on whether you cancel 1st)

$$9x - 6 = 48x - 30$$

$-48x$ $+6$ $-48x$ $+6$

$$9x - 48x = -30 + 6$$

$$-39x = -24$$

$$\frac{-39x}{-39} = \frac{-24}{-39}$$

$$x = \frac{24}{39}$$

$$x = \frac{24 \div 3}{39 \div 3} = \frac{8}{13}$$

Solution set: $\left\{\frac{8}{13}\right\}$

(see divisibility tip on next page)

Tip: If the sum of the digits of a number is divisible by 3 (or 9), then the number is divisible by 3 (or 9)

Example: $\frac{1}{4}x - \frac{2}{5}x + \frac{1}{3} - \frac{5}{2} = 6x - \frac{2}{3}$

Multiply both sides by 60:

$$60\left(\frac{1}{4}x\right) + 60\left(-\frac{2}{5}x\right) + 60\left(\frac{1}{3}\right) - 60\left(\frac{5}{2}\right) = 60(6x) - 60\left(\frac{2}{3}\right)$$

(depending on whether you cancel before or after multiplying)

$$\frac{60}{4}x - \frac{120}{5}x + \frac{60}{3} - \frac{300}{2} = 360x - \frac{120}{3}$$

$$15x - 24x + 20 - 150 = 360x - 40$$

$$-9x - 130 = 360x - 40$$

$$-9x - 360x = -40 + 130$$

$$-369x = 90$$

$$\frac{-369x}{-369} = \frac{90}{-369}$$

$$x = -\frac{90}{369}$$

Reduce if!

$$x = -\frac{90 \div 9}{369 \div 9} = -\frac{10}{41}$$

Soln Set: $\left\{-\frac{10}{41}\right\}$

$3+6+9 = 18$, which is divisible by 9, so 369 is divisible by 9:

$$9\overline{)369}\quad 41,\ 36,\ 09,\ 9,\ 0$$

Check: $41 \times 9 = 369$

5. $\frac{1}{12}(9a-3)=\frac{1}{4}(3a-1)$

$\frac{1}{12}(9a)+\frac{1}{12}(-3)=\frac{1}{4}(3a)+\frac{1}{4}(-1)$

$\frac{9a}{12}-\frac{3}{12}=\frac{3a}{4}-\frac{1}{4}$

$\frac{3}{4}a-\frac{1}{4}=\frac{3}{4}a-\frac{1}{4}$

Both sides are the same! This is an identity (true for every value of the variable

Soln Set: All real numbers

6. $\frac{5a-3}{10}=2-\frac{a-3}{5}$

$\frac{5a}{10}-\frac{3}{10}=2-\left(\frac{a}{5}-\frac{3}{5}\right)$

$\frac{1}{2}a-\frac{3}{10}=2-\frac{a}{5}+\frac{3}{5}$

multiply by 10:

$10\left(\frac{1}{2}a\right)-\frac{3}{10}(10)=2(10)-\frac{a}{5}(10)+\frac{3}{5}(10)$

$5a-3=20-2a+6$

$5a-3=26-2a$

$5a+2a=26+3$

$7a=29$

$\frac{7a}{7}=\frac{29}{7}$

$a=\frac{29}{7}$

$\left\{\frac{29}{7}\right\}$

Linear Equations Containing Decimals

The same procedure used to clear fractions in an equation can be used to clear decimals.

To clear decimals: clear parentheses first, then
multiply all terms by the appropriate power of 10.

Tip: Multiplying a decimal number by 10 has the affect of moving the decimal point one place to the right. Similarly, multiplying by 100 moves the decimal point two places to the right, and so on.

Determine which of the values could be used to clear decimals in the given equation.

7. $-0.2x+6=0.35x+4.5$

Values: 10; 100; 1000; 10,000

8. $0.4-0.005x=x$

Values: 10; 100; 1000; 10,000

Solve the equation.

9. $-0.2x + 53.6 = x$

Multiply both sides by 10:

$10(-0.2x + 53.6) = 10(x)$

$-2x + 536 = 10x$

$+2x \quad +2x$

$536 = 12x$

$\frac{536}{12} = \frac{12x}{12}$ or $x = 44\frac{8}{12}$

$\frac{134}{3} = x$ $= 44\frac{2}{3}$

$\left\{\frac{134}{3}\right\}$ $\left\{44\frac{2}{3}\right\}$

$12\overline{)536}$ $\quad 44\frac{8}{12}$

$4\overline{)536}$ $\quad 134$

10. $0.75(m-2) + 0.25m = 0.5$

Multiply both sides by 100:

$100(0.75)(m-2) + 100(0.25m) = 100(0.5)$

$75(m-2) + 25m = 50$

$75m - 150 + 25m = 50$

$100m - 150 = 50$

$+150 \quad +150$

$100m = 200$

$\frac{100m}{100} = \frac{200}{100}$

$m = 2$

$\{2\}$

11. $0.7(a-1) = 0.25 + 0.7a$

12. $0.08(6-x) = 0.02(x-26)$

13. $0.5x + 0.25 = \frac{1}{3}x + \frac{5}{4}$

Homework Qs

2.1) #43 $\frac{4}{5} = -t$

What happens if we multiply by $\frac{5}{1}$?

$$\left(\frac{5}{1}\right)\frac{4}{5} = -\frac{t}{1}\left(\frac{5}{1}\right)$$

$$4 = -5t$$

Divide by -5: $\frac{4}{-5} = \frac{-5t}{-5}$

$$-\frac{4}{5} = t$$

$$\left\{-\frac{4}{5}\right\}$$

More efficient way:

$$\frac{4}{5} = -t$$

$$\frac{4}{5} = -1t$$

Goal: isolate the t (Get it on one side)

Multiply by -1:

$$\frac{4}{5}(-1) = -1t\,(-1)$$

$$-\frac{4}{5} = 1t$$

$$-\frac{4}{5} = t$$

Sol'n Set: $\left\{-\frac{4}{5}\right\}$

2.1 #47 $-\frac{1}{5}b = -\frac{4}{5}$

Multiply both sides by $-\frac{5}{1}$:

$$-\frac{5}{1}\left(-\frac{1}{5}b\right) = \left(-\frac{4}{5}\right)\left(-\frac{5}{1}\right)$$

$$b = \frac{20}{5}$$

$$b = 4$$

Solution set: $\{4\}$

2.1 #51 $3.81 = -0.03p$

Divide by -0.03:

$$\frac{3.81}{-0.03} = \frac{-0.03p}{-0.03}$$

$$p = \frac{3.81}{-0.03} = -127$$

$$\begin{array}{r} 127 \\ .03\overline{)3.81} \\ 3 \\ \hline 8 \\ 6 \\ \hline 21 \end{array}$$

Trick: $\frac{3.81}{-0.03}\left(\frac{100}{100}\right)$

$$= \frac{381}{-3} = -127$$

HW Qs from Tues 9/13

2.3 #44 $\quad z + 0.06z = 3816$

Multiply by 100:

$$100z + 100(0.06z) = 3816(100)$$

$$100z + 6z = 381600$$

$$106z = 381600$$

$$\frac{106z}{106} = \frac{381600}{106}$$

$$z = \frac{190800}{53} = 3600$$

$\{3600\}$

$$\begin{array}{r} 190800 \\ 2\overline{)381600} \\ \underline{2} \\ 18 \\ \underline{18} \\ 016 \\ \underline{16} \\ 00 \\ \vdots \\ 190800 \\ \hline 2\overline{)381600} \end{array}$$

2.3 #23

$$\frac{1}{4}(3m-4) - \frac{1}{5} = \frac{1}{4}m + \frac{3}{10}$$

$$\frac{1}{4}(3m) + \frac{1}{4}(-4) - \frac{1}{5} = \frac{1}{4}m + \frac{3}{10}$$

$$\frac{3}{4}m - 1 - \frac{1}{5} = \frac{1}{4}m + \frac{3}{10}$$

mult. by 20:

$$20\left(\frac{3}{4}m\right) - 1(20) - \frac{1}{5}(20) = \frac{1}{4}m(20) + \frac{3}{10}(20)$$

$$15m - 20 - 4 = 5m + 6$$

$$15m - 24 = 5m + 6$$

$$-5m \quad +24 \qquad -5m \quad +24$$

$$10m = 30$$

$$\frac{10m}{10} = \frac{30}{10}$$

$m = 3$

$\{3\}$