Problem-Solving Strategies

Step 1: Read the problem carefully.	Familiarize yourself with the problem. Identify the unknown, and if possible, estimate the answer.
Step 2: Assign labels to unknown quantities.	Identify the unknown quantity or quantities. Let a variable represent one of the unknowns. Draw a picture and write down relevant formulas.
Step 3: Write a verbal model.	Write an equation in *words*.
Step 4: Write a mathematical equation.	Replace the verbal model with a mathematical equation using x or another variable.
Step 5: Solve the equation.	Solve for the variable using the steps for solving linear equations.
Step 6: Interpret the results and write the final answer in words.	Once you've obtained a numerical value for the variable, recall what it represents in the context of the problem. Can this value be used to determine other unknowns in the problem? Write an answer to the word problem in *words*.

Translations Involving Linear equations

Write an expression representing the unknown quantity.

1. Donna is 5 inches taller than Cristina. If x represents Cristina's height in inches, write an expression for Donna's height.

Cristina's height: x

Donna's height: $x+5$

2. John's test grade is 5 points less than twice Ariana's test grade. If x represents Ariana's test grade, write an expression for John's test grade.

$2x$

Ariana's grade: x

John's grade: $2x-5$

Use the problem-solving flowchart.

3. Forty more than a number is twelve. Find the number.

Step 2: "A number": x

Step 3: Write eqn:

$$x + 40 = 12$$

Step 4: $-40 \quad -40$

Step 5:

$$x = -28$$

Answer in a complete sentence:

Step 6: Check: $-28 + 40 = 12$ ✓

The number is -28.

4. The difference of twice a number and 8 is twenty. Find the number.

The number: x

$$2x - 8 = 20$$

$$2x = 28$$

$$\frac{2x}{2} = \frac{28}{2}$$

$$x = 14$$

The number is 14.

Check it:
Twice the number: 28
Subtract 8: 20 ✓

5. Twice the difference of a number and 8 is twenty. Find the number.

The number: x

$$2(x - 8) = 20$$

$$2x - 16 = 20$$

$+16 \quad +16$

$$2x = 36$$

$$\frac{2x}{2} = \frac{36}{2}$$

$$x = 18$$

The number is 18.

Check it:
Diff of number and 8
$18 - 8 = 10$
Twice the difference:
20 ✓

$x-6$

6. If six less than a number is tripled, the result is five more than the number. Find the number.

The number: x

$$3(x-6) = x+5$$
$$3x-18 = x+5$$
$$3x = x+23$$
$$2x = 23$$
$$\frac{2x}{2} = \frac{23}{2}$$
$$x = \frac{23}{2} = 11\frac{1}{2}$$

The number is $\frac{23}{2}$.

$2x$

7. Four times a number is the same as the sum of twice a number and ten. Find the number.

The number: x

$$4x = 2x+10$$
$$2x = 10$$
$$x = 5$$

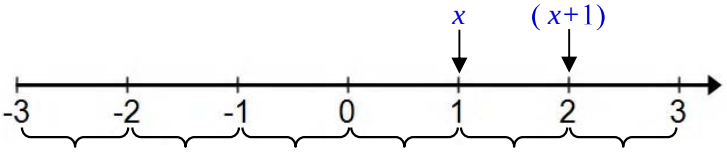
The number is 5

Consecutive Integer Problems

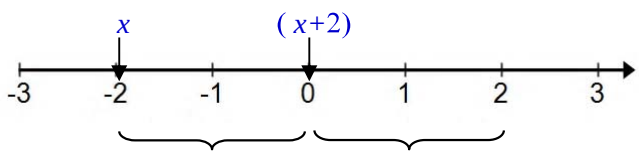
The word *consecutive* means "following one after the other in order without gaps."

1st integer: x
2nd integer: $x+1$
3rd integer: $x+2$
etc

Consecutive integers differ by 1 unit

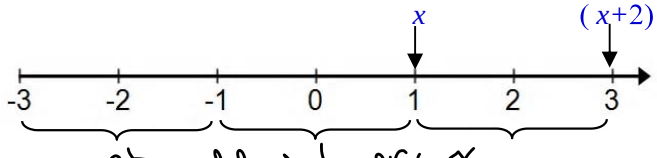


Consecutive even integers differ by 2 units



1st even integer: x
2nd even integer: $x+2$
3rd: $x+4$

Consecutive odd integers differ by 2 units



1st odd integer: x
2nd odd integer: $x+2$
3rd odd integer: $x+4$

8. The sum of two consecutive odd integers is -172. Find the integers.

Step 2: 1st odd integer: x

2nd odd integer: $x+2$

Step 3:

$$x+(x+2)=-172$$

Step 4:

$$x+x+2=-172$$

Step 5:

$$2x+2=-172$$

$$2x=-174$$

$$\frac{2x}{2}=\frac{-174}{2}$$

$$x=-87$$

$$2\overline{)174}\quad 87,\ 16,\ 14$$

Step 6:

1st integer: $x=-87$

Plug in for x: $x=-87$

2nd odd integer: $x+2=-87+2=-85$

The consecutive odd integers are -87 and -85.

9. Five times the sum of three consecutive integers is 1 less than 14 times the largest integer. Find the integers.

smallest → 1st integer: x

2nd integer: $x+1$

3rd integer: $x+2$ → largest

5(sum of integers) = 14(largest) − 1

$$5(x+(x+1)+(x+2))=14(x+2)-1$$

$$5(x+x+1+x+2)=14(x+2)-1$$

$$5(3x+3)=14(x+2)-1$$

$$15x+15=14x+28-1$$

$$15x+15=14x+27$$

$-14x \quad -14x$

$$x+15=27$$

$$x=27-15$$

$$x=12$$

1st integer: $x=12$

2nd integer: $x+1$

$12+1$

13

3rd integer: $x+2$

$12+2$

14

The integers are 12, 13, 14.

Recall: Consecutive even integers: $x, x+2, x+4, x+6, \ldots$

Consecutive odd integers: $x, x+2, x+4 \ldots$

consecutive integers: $x, x+1, x+2, \ldots$

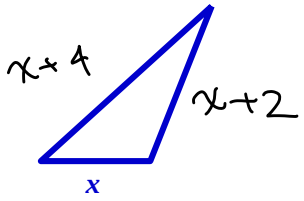
10. The perimeter of a triangle is 24 ft. The lengths of the sides are represented by consecutive even integers. Find the measures of the sides.

1st side: x

another side: $x+2$

3rd side: $x+4$

Perimeter = sum of all side lengths



$24 = x + (x+2) + (x+4)$

$24 = x + x + 2 + x + 4$

$24 = 3x + 6$

$-6 \quad -6$

$18 = 3x$

$\frac{18}{3} = \frac{3x}{3}$

$6 = x$

1st side: $x = 6$

2nd side: $x+2 = 6+2 = 8$

3rd side: $x+4 = 6+4 = 10$

The sides are 6ft, 8ft, and 10ft long.

Applications of Linear Equations

11. A pvc pipe is 125 in. in length and must be cut so that one piece is 15 in. longer than the other piece. Find the length of each piece.

Step 2:

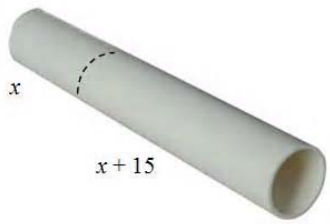
One piece: $x+15$

other piece: x

Step 3:

Write an equ:

$125 = (x+15) + x$



Step 4:

$125 = x + 15 + x$

Step 5:

$125 = 2x + 15$

$110 = 2x$

$\frac{110}{2} = \frac{2x}{2}$

$x = 55$

other piece: $x = 55$

one piece: $x+15 = 55+15 = 70$

Step 6:

The pieces are 55 inches and 70 inches.

12. In June 2011, Justin Beiber had earned $17 million less than Lady Gaga earned over a 12-month period. The total earned by the two artists over this time period was $123 million. Determine the amount of money Justin Bieber earned in this 12-month period.

Amt Justin made: $x-17$
Amount Gaga made: x
(amounts in milliars)

Justin compared to Gaga
$x-17$ → x

$$x + x - 17 = 123$$
$$2x - 17 = 123$$
$$+17 \quad +17$$
$$2x = 140$$
$$\frac{2x}{2} = \frac{140}{2}$$
$$x = 70$$

Gaga's income: $x=70$
Justin's Income:
$x-17$
$=70-17=53$

Justin made $53 million and Gaga made $70 million.

13. The circumference of Earth at the equator is 5851 miles more than twice the circumference of Mercury at the equator. If the sum of their circumferences is 34, 426 miles, find the circumference of each planet.