Writing an Equation of a Line Using the Point-Slope Formula

Point -Slope Formula: The point-slope formula is given by

$$y - y_1 = m(x - x_1)$$

Note: Dividing by $x - x_1$ results in $\frac{y - y_1}{x - x_1} = m$

where m is the slope of the line and (x_1, y_1) is a known point on the line.

For exercises $1 - 4$, use the point-slope formula (if possible) to write an equation of the line given the following information.

1. The slope is 5, and the line passes through the point $(-4, 3)$.

$m = 5$

$(x_1, y_1) = (-4, 3)$

$x_1 = -4$

$y_1 = 3$

$y - y_1 = m(x - x_1)$

$y - 3 = 5(x - (-4))$ Pt-slope form

Put in slope-intercept form:

$y - 3 = 5(x + 4)$

$y - 3 = 5x + 20$

$+3 \quad +3$

$y = 5x + 23$

2. The slope is $-\frac{2}{3}$, and the line passes through the point $(6, -4)$.

$m = -\frac{2}{3}$

$x_1 = 6$

$y_1 = -4$

$y - y_1 = m(x - x_1)$

$y - (-4) = -\frac{2}{3}(x - 6)$

$y + 4 = -\frac{2}{3}x - \frac{2}{3}(-6)$

$y + 4 = -\frac{2}{3}x + \frac{12}{3}$

$y + 4 = -\frac{2}{3}x + 4$

$-4 \quad -4$

$y = -\frac{2}{3}x$

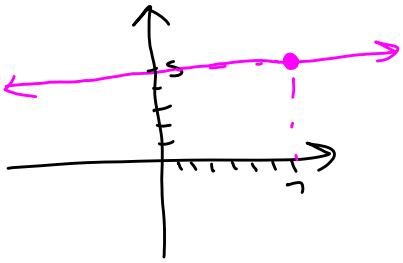
Note: y-intercept is 0

$y = -\frac{2}{3}x + 0$

3. The slope is 0, and the line passes through the point $(7, 5)$.

Sketch it:

line must be horizontal



Equation of this line is $y = 5$

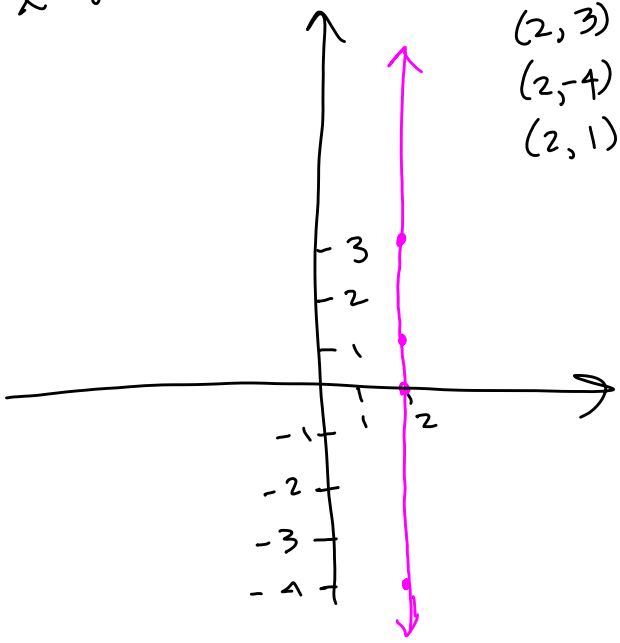
See next page

4. The slope is 4.1, and the line passes through the point $(-0.5, -1.2)$.

Writing an Equation of a Line Given Two Points

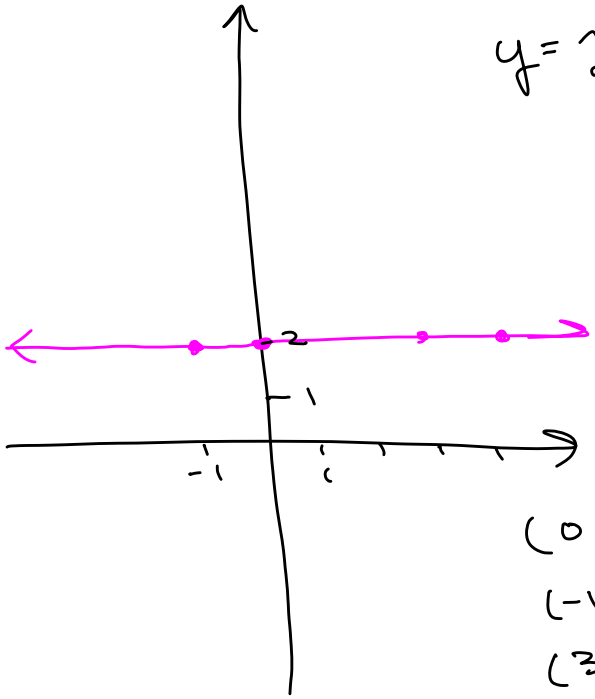
If two points are given: First find the slope from the two given points, then substitute the slope and one point into the point-slope formula.

$x = 2$

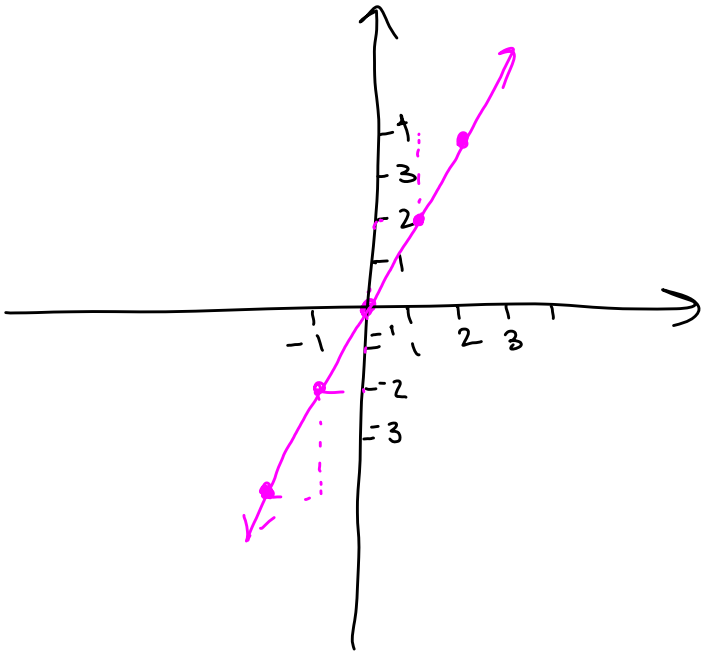


$(2, 0)$
$(2, 3)$
$(2, -4)$
$(2, 1)$

$y = 2$



$(0, 2)$
$(-1, 2)$
$(3, 2)$
$(4, 2)$



$y = 2x$

Slope-intercept form: $y = mx + b$

$y = 2x + 0$

y-intercept: $b = 0$

slope: $m = 2 = +\frac{2}{1}$

x-intercept: 0

y-intercept: 0

Find x-intercept: set $y = 0$.

$y = 2x$

$0 = 2x$

$\frac{0}{2} = \frac{2x}{2}$

$0 = x$

For exercises 5 and 6, use the point-slope formula to write an equation of the line given the following information.

5. The line passes through the points $(-2,3)$ $(-6,15)$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15-3}{-6-(-2)} = \frac{12}{-6+2} = \frac{12}{-4} = -3$

$y - y_1 = m(x - x_1)$

using $m = -3$, $x_1 = -2$, $y_1 = 3$

$(x_1, y_1) = (-2, 3)$

$y - 3 = -3(x-(-2))$ ← pt-slope form

$y - 3 = -3(x+2)$

$y - 3 = -3x - 6$

$+3 \quad +3$

$y = -3x - 3$ slope-intercept form

6. The line passes through the points $(-0.8,-3.5)$ and and $(2.2,5.5)$.

Alternative method for finding eqn of line in #5:

still use $m = -3$.

Choose one of the points: $(x, y) = (-2, 3)$

Put into $y = mx + b$:

$m = -3 \Rightarrow y = -3x + b$

Use $x = -2$, $y = 3$ to find b:

$3 = -3(-2) + b$

$3 = 6 + b$

$-6 \quad -6$

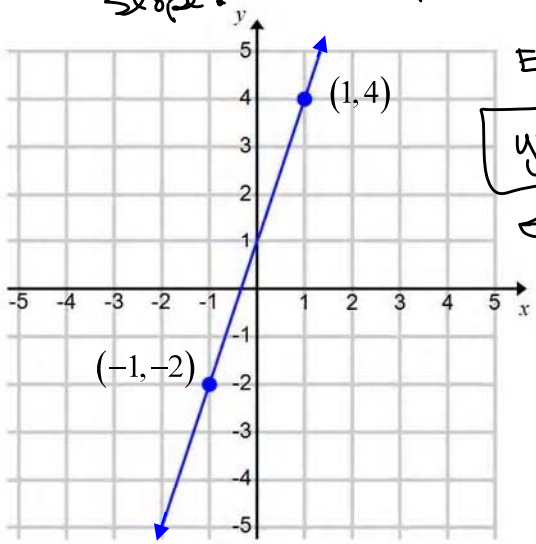
$-3 = b$. Then $y = -3x - 3$

For exercises 7 and 8, find an equation of the line through the given points. Write the final answer in slope-intercept form.

7.

y-intercept: $(0, 1)$, so $b = 1$

slope: $m = +\frac{3}{1} = 3$

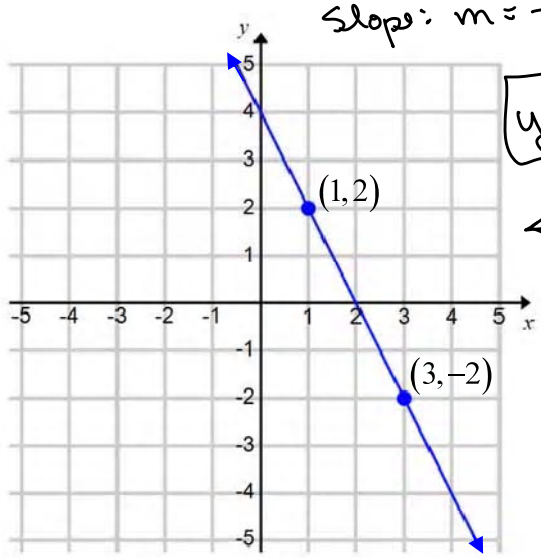


Equation: $y = 3x + 1$

8.

y-intercept: $(0,4)$, so $b = 4$

slope: $m = -\frac{2}{1} = -2$



$y = -2x + 4$

Recall: * slopes of parallel lines are equal
* slopes of perpendicular lines are opposite reciprocals

Writing an Equation of a Line Parallel or Perpendicular to Another Line

For exercises 9 and 10, use the point-slope formula to write an equation of the line given the following information.

9. The line passes through the point $(-4,3)$ and is perpendicular to the line $y=-\frac{1}{2}x+5$.

Find slope of given line:

$y=-\frac{1}{2}x+5$

$m_1=-\frac{1}{2}$

Slope of perpendicular line:

$m=+\frac{2}{1}=2$

$y-y_1=m(x-x_1)$

$x_1=-4,\ y_1=3,\ m=2$ give us

$y-3=2(x-(-4))$

$y-3=2(x+4)$

$y-3=2x+8$

$y=2x+11$

10. The line passes through the point $(6,4)$ and is parallel to the line $4x-y=7$.

Find slope of given line.

Write as $y=mx+b$:

$4x-y=7$

$-y=-4x+7$

$\frac{-y}{-1}=\frac{-4x}{-1}+\frac{7}{-1}$

$y=4x-7$

$y=4x-7$

$m_1=4=+\frac{4}{1}$

Slope of parallel line is $m=4$

$y-y_1=m(x-x_1)$ & $\begin{cases}x_1=6\\y_1=4\end{cases}$

$y-4=4(x-6)$

$y-4=4x-24$

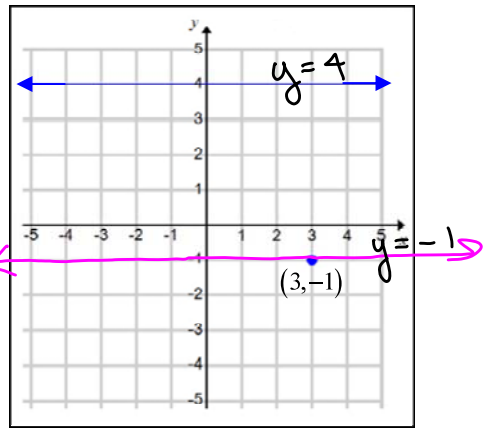
$y=4x-20$

Different Forms of Linear Equation: A Summary

A linear equation can be written in several different forms, as summarized in the table below.

Form	Example	Comments
Standard Form $Ax+By=C$	$2x-3y=5$	A and B must not both be zero.
Horizontal Line $y=k$ (k is constant)	$y=-3$	The slope is zero, and the y-intercept is $(0,k)$.
Vertical Line $x=k$ (k is constant)	$x=-1$	The slope is undefined, and the x-intercept is $(k,0)$.
Slope-Intercept Form $y=mx+b$ the slope is m y-intercept $(0,b)$	$y=-5x-6$ Slope $=-5$ y-intercept is $(0,-6)$	Solving a linear equation for y results in slope-intercept form. The coefficient of the x-term is the slope, and the constant defines the location of the y-intercept.
Point-Slope Formula $y-y_1=m(x-x_1)$	$m=5$ $(x_1,y_1)=(-1,4)$ $y-4=5(x+1)$	The formula is typically used to build an equation of a line when a point on the line is known and the slope of the line is known.

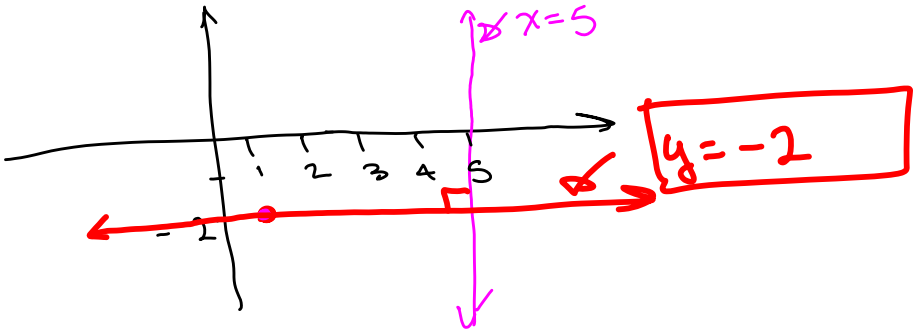
11. Find an equation for the line given the following information.
The line passes through the point $(3,-1)$ and is parallel to the line $y=4$. See the figure.



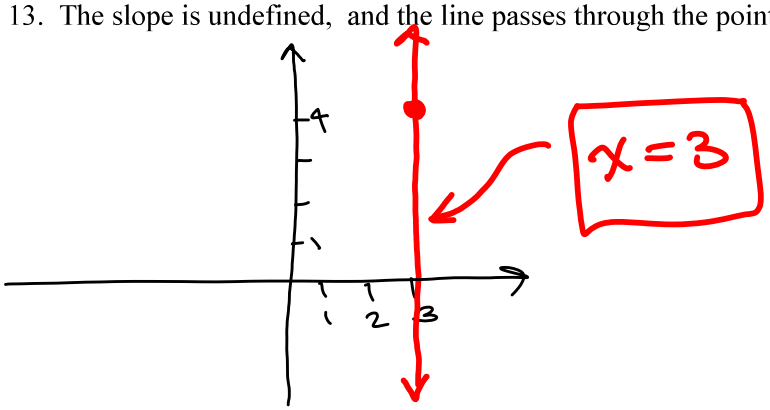
$y=-1$

For exercises 12 – 14, find an equation for the line given the following information.

12. The line passes through the point $(1,-2)$ and is perpendicular to the line $x=5$. (*Hint:* Sketch the line first.)



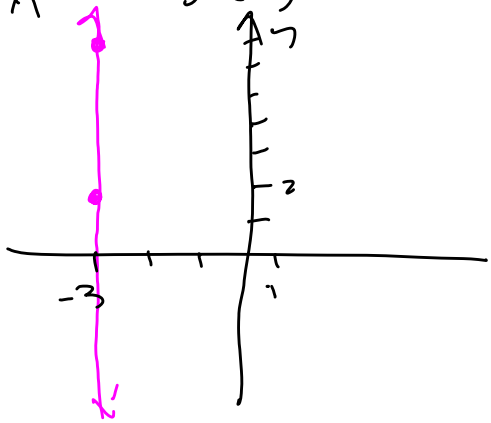
13. The slope is undefined, and the line passes through the point $(3,4)$.



14. The line passes through the point $(-3,7)$ and $(-3,2)$.

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{2-7}{-3-(-3)} = \frac{-5}{-3+3} = \frac{-5}{0}$ No! it's undefined!

slope is undefined



Equation of line:

$x=-3$

HW Qs

3.4 #50: Write in slope-intercept form and graph.

$$6x = 2y - 14$$

Slope-intercept form: $y = mx + b$

$$6x = 2y - 14$$

$$2y - 14 = 6x$$

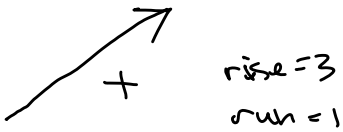
$+14 \quad +14$

$$2y = 6x + 14$$

$$\frac{2y}{2} = \frac{6x}{2} + \frac{14}{2}$$

$$y = 3x + 7$$

Slope: $m = 3 = +\frac{3}{1}$



y-intercept: $b = 7$