Definition of b^0

Definition of b^0 Let b be a nonzero real number. Then, $b^0 = 1$.

why?

(assuming variables represent nonzero numbers)

For exercises 1 – 6, simplify the expressions.

1. $(-3)^0$

$= \boxed{1}$

2. -3^0

$-1 \cdot 3^0$

$= -1 \cdot 1 = \boxed{-1}$

3. $(-6p)^0$

$\boxed{1}$

(assuming $p \neq 0$)

$\frac{8}{8} = 1$

$\frac{2^3}{2^3} = 2^{3-3} = 2^0$

$\frac{32}{32} = 1$

$\frac{2^5}{2^5} = 2^{5-5}$

$= 2^0$

4. $-6p^0$

$-6 \cdot p^0$

$= -6(1)$

$= \boxed{-6}$

5. p^0

$= \boxed{1}$

6. $\frac{p^5}{p^5}$

$= p^{5-5}$

$= p^0 = \boxed{1}$

Definition of b^{-n}

Also $\frac{1}{b^{-n}} = \frac{b^n}{1}$

Definition of b^{-n} Let n be an integer and b be a nonzero real number. Then,

$b^{-n} = \frac{1}{b^n}$

$\cancel{b^{-n} = \left(\frac{1}{b}\right)^n \text{ or } \frac{1}{b^n}}$

$b^{-n} = \frac{b^{-n}}{1} = \frac{1}{b^n}$

Note: To evaluate b^{-n}, take the reciprocal of the base and change the sign of the exponent.

For exercises 7 – 12, simplify.

7. $\left(\frac{5}{6}\right)^{-3}$

$\left(\frac{5}{6}\right)^{-3} = \frac{5^{-3}}{6^{-3}} = \frac{6^3}{5^3}$

$= \boxed{\frac{216}{125}}$

8. $\left(-\frac{2}{3}\right)^{-4}$

$\left(-\frac{2}{3}\right)^{-4}$

$= \left(\frac{-2}{3}\right)^{-4}$

$= \frac{(-2)^{-4}}{(3)^{-4}}$

$= \frac{(3)^4}{(-2)^4} = \boxed{\frac{81}{16}}$

9. $(d)^{-7}$

$\frac{d^{-7}}{1} = \boxed{\frac{1}{d^7}}$

$$3^4 = 81$$
$$3^3 = 81 \div 3 = 27$$
$$3^2 = 27 \div 3 = 9$$
$$3^1 = 9 \div 3 = 3$$
$$3^0 = 3 \div 3 = 1$$

Ex: $\left(\dfrac{2x}{3}\right)^{-4}$

$$= \frac{2^{-4}x^{-4}}{3^{-4}}$$

$$= \frac{3^4}{2^4 x^4}$$

$$= \boxed{\frac{81}{16x^4}}$$

Ex: $(4a^3b^{-2})^4$

$$= 4^4 a^{12} b^{-8}$$

$$= \frac{4^4 a^{12} b^{-8}}{1}$$

$$= \boxed{\frac{256a^{12}}{b^8}}$$

In this class, we will write all variables with positive exponents only.

Ex: $\dfrac{4(3x^{-1}y^2z^{-3})^{-1}}{(5y^{-3})^2}$

$$= \frac{4 \cdot 3^{-1} x^1 y^{-2} z^3}{5^2 y^{-6}}$$

$$= \frac{4}{3^1} \frac{x}{} \frac{z^3}{y^2} \frac{y^6}{\cdot 5^2}$$

$$= \frac{4xy^6z^3}{3 \cdot 25 y^2} = \boxed{\frac{4xy^4z^3}{75}}$$

Ex: $\dfrac{x^{-3}y^4z}{x^{-8}y^9z^{-2}}$

$$= \frac{y^4 z x^8}{x^3} \frac{z^2}{y^9}$$

$$= \frac{x^8 y^4 z^3}{x^3 y^9}$$

$$= \boxed{\frac{x^5 z^3}{y^5}}$$

10. $(6x)^{-2}$

$= 6^{-2} x^{-2}$

$= \frac{6^{-2} x^{-2}}{1}$

$= \frac{1}{6^2 x^2} = \boxed{\frac{1}{36x^2}}$

11. $6x^{-2}$

$\frac{6x^{-2}}{1}$

$= \boxed{\frac{6}{x^2}}$

12. $\frac{1}{n^{-8}}$

$= \frac{n^8}{1}$

$= \boxed{n^8}$

13. Simplify and write the answers with positive exponents.

a. a^{-9}

$\frac{a^{-9}}{1} = \boxed{\frac{1}{a^9}}$

b. $\frac{a^2}{a^{11}}$ $= \boxed{\frac{1}{a^9}}$

Note: $\frac{a^2}{a^{11}} = a^{2-11} = a^{-9} = \frac{1}{a}$

Properties of Integer Exponents: A Summary

Properties of Integer Exponents Assume that a and b are real numbers $(b \neq 0)$ and that m and n represent integers.		
Property	**Example**	**Detail/Notes**
Multiplication of Expressions with Like Bases $b^m b^n = b^{m+n}$	$b^2 b^6 = b^{2+6} = b^8$	$b^2 b^6 = (b \cdot b)(b \cdot b \cdot b \cdot b \cdot b \cdot b) = b^8$
Division of Expressions with Like Bases $\frac{b^m}{b^n} = b^{m-n}$	$\frac{b^7}{b^4} = b^{7-4} = b^3$	$\frac{b^7}{b^4} = \frac{\cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot b \cdot b \cdot b}{\cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b}} = b^3$
The Power Rule $(b^m)^n = b^{m \cdot n}$	$(b^3)^2 = b^{3 \cdot 2} = b^6$	$(b^3)^2 = (b \cdot b \cdot b)(b \cdot b \cdot b) = b^6$
Power of a Product $(ab)^m = a^m b^m$	$(ab)^4 = a^4 b^4$	$(ab)^4 = (ab)(ab)(ab)(ab)$ $= (a \cdot a \cdot a \cdot a)(b \cdot b \cdot b \cdot b) = a^4 b^4$
Power of a Quotient $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$	$\left(\frac{a}{b}\right)^3 = \left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) = \frac{a \cdot a \cdot a}{b \cdot b \cdot b} = \frac{a^3}{b^3}$
Definitions Assume that b is a real number $(b \neq 0)$ and that n represents an integer.		
Definition	**Example**	**Details/Notes**
$b^0 = 1$	$(3)^0 = 1$	Any nonzero quantity raised to the zero power equals 1.
$b^{-n} = \left(\frac{1}{b}\right)^n = \frac{1}{b^n}$	$b^{-4} = \left(\frac{1}{b}\right)^4 = \frac{1}{b^4}$	To simplify a negative exponent, take the reciprocal of the base and make the exponent positive.

For exercises 14 – 24, simplify the expression. Write the answer with positive exponents only.

14. $a^{-5}a^{3}$

$a^{-5}a^{3}$

15. $\left(r^{3}s^{-4}\right)^{-5}$

$\left(r^{3}s^{-4}\right)^{-5}$

16. $\dfrac{w^{4}}{w^{10}}$

$\dfrac{w^{4}}{w^{10}}$

.

17. $\dfrac{t^{2}}{t^{-8}}$

$\dfrac{t^{2}}{t^{-8}} = \dfrac{t^{2}\,t^{8}}{1}$

$= \dfrac{t^{10}}{1} = \boxed{t^{10}}$

18. $\dfrac{5^{3}}{5^{2}\cdot 5^{4}}$

$\dfrac{5^{3}}{5^{2}\cdot 5^{4}}$

$= \dfrac{5^{3}}{5^{6}}$

$= \dfrac{1}{5^{3}} = \boxed{\dfrac{1}{125}}$

19. $\dfrac{a^{-2}\left(a^{2}\right)^{-3}}{a^{4}}$

$\dfrac{a^{-2}\left(a^{2}\right)^{3}}{a^{4}}$

20. $\left(\dfrac{b^{2}c^{-5}}{d^{-4}}\right)^{0}$

$\left(\dfrac{b^{2}c^{-5}}{d^{-4}}\right)^{0} = \boxed{1}$

21. $\left(4x^{2}y^{0}\right)^{-2}$

$\left(4x^{2}y^{0}\right)^{-2}$

$4^{-2}\,x^{-4}\,y^{0}$

$= \dfrac{y^{0}}{4^{2}x^{4}}$

$= \boxed{\dfrac{1}{16x^{4}}}$

22. $\dfrac{-20s^{12}t^{7}}{45s^{-6}t^{7}}$

$$\frac{-\cancel{20}^{\,4}\, s^{12} t^{7}}{\cancel{45}_{\,9}\, s^{-6} t^{7}}$$

$$= \frac{-4 s^{12} t^{7}}{9} \cdot \frac{s^{6}}{t^{7}}$$

$$= \frac{-4 s^{18} \cancel{t^{7}}^{\,1}}{9 \cancel{t^{7}}_{\,1}}$$

$$= \boxed{-\frac{4s^{18}}{9}}$$

23. $\left(3a^{3}b\right)\left(\dfrac{8ab^{2}}{9a^{5}b^{3}}\right)$

$$\frac{\left(3a^{3}b\right)}{1}\left(\frac{8ab^{2}}{9a^{5}b^{3}}\right)$$

$$= \frac{3a^{3}b \cdot 8ab^{2}}{9a^{5}b^{3}}$$

$$= \frac{\cancel{24}^{\,8}\, a^{4} \cancel{b^{3}}^{\,1}}{\cancel{9}_{\,3}\, a^{5} \cancel{b^{3}}_{\,1}}$$

$$= \frac{8a^{4}}{3a^{5}}$$

$$= \boxed{\frac{8}{3a}}$$

24. Simplify the expression.

$2^{-1} + 3 \cdot 8^{-1}$

$$2^{-1} + 3 \cdot 8^{-1}$$

$$\frac{2^{-1}}{1} + \frac{3 \cdot 8^{-1}}{1}$$

$$= \frac{1}{2^{1}} + \frac{3}{8^{1}}$$

$$= \frac{1}{2} + \frac{3}{8}$$

$$= \frac{1}{2} \cdot \frac{4}{4} + \frac{3}{8} = \frac{4}{8} + \frac{3}{8} = \boxed{\frac{7}{8}}$$