Addition and Subtraction of Polynomials

Section 5.5

Introduction to Polynomials

A polynomial in one variable, x, is defined as a single term or a sum of terms of the form ax^n, where a is a real number and the exponent, n, is a nonnegative integer.

Polynomials: x^2+17
$4x^3-12x^9+3$
$\frac{1}{2}x^4-x^2+7$

For each term ax^n, a is called the coefficient, and n is called the degree of the term.

Not polynomials:
$\frac{1}{x^2}+\frac{5}{x^3}$, $3\sqrt{x}+x^2$
negative exponents

A monomial is a polynomial that has exactly one term.
$3x^5$ is a monomial

A binomial is a polynomial that has exactly two terms.
$4x^2-7x$ is a binomial

A trinomial is a polynomial that has exactly three terms.
x^2+5x+2 is a trinomial

The term with highest degree is called the leading term, and its coefficient is called the leading coefficient.

The degree of a polynomial is the greatest degree of all of its terms.
$4x^3-12x^9+3$ is a 9th degree trinomial. Leading coefficient is -12.

If a polynomial has more than one variable, the degree of a term is the sum of the exponents of the variables contained in the term.

Note: The terms of a polynomial are usually written in descending order according to degree.

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1. Write the polynomial in descending order: $8-10c^5+8c^2-c^3$

$-10c^5-c^3+8c^2+8$

Leading term: $-10c^5$
Leading coefficient: -10

For exercises 2 – 4, categorize the expression as a monomial, a binomial, or a trinomial. Then identify the coefficient and degree of the leading term.

2. $5c-c^3$

binomial
3rd degree
leading coefficient: -1
could write as: $-1c^3+5c$

3. $7a^2b^3c$

monomial
Degree: $2+3+1=6$
Leading coefficient: 7

4. $6y-5+3y^2$

trinomial
Leading term: $3y^2$
Leading coefficient: 3
Degree: 2
Rearrange: $3y^2+6y-5$

Addition of Polynomials

Two terms are *like* terms if they each have the same variables, and the corresponding variables are raised to the same powers.

To add polynomials: Group *like* terms.
Combine the *like* terms by adding their coefficients.

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5. Explain why the terms $5a$ and $5a^3$ are not *like* terms.

exponents don't match

For exercises 6 – 9, add the polynomials.

6. $(5a+3b)+(3a-7b)$

$= 5a+3b+3a-7b$

$= \boxed{8a-4b}$

7. $(4x^2+8x-6)+(-6x^2-x+9)$

$4x^2+8x-6-6x^2-x+9$

$= \boxed{-2x^2+7x+3}$

8. $\left(\frac{3}{4}c+\frac{1}{5}d-\frac{3}{8}\right)+\left(\frac{1}{4}c+\frac{2}{5}d-\frac{1}{8}\right)$

$\frac{3}{4}c+\frac{1}{5}d-\frac{3}{8}+\frac{1}{4}c+\frac{2}{5}d-\frac{1}{8}$

$= \frac{3}{4}c+\frac{1}{4}c+\frac{1}{5}d+\frac{2}{5}d-\frac{3}{8}-\frac{1}{8}$

$= \frac{4}{4}c+\frac{3}{5}d-\frac{4}{8}$

$= 1c+\frac{3}{5}d-\frac{1}{2}$

$= \boxed{c+\frac{3}{5}d-\frac{1}{2}}$

9.
$$\begin{array}{rrrr} & 0.23g^3 & & +1.2g-6 \\ + & & -3.4g^2 & +2.4g-2 \\ \hline \end{array}$$

$\boxed{0.23g^3-3.4g^2+3.6g-8}$

Subtraction of Polynomials

Opposite of a Polynomial: To find the opposite of a polynomial, take the opposite of each term. This is equivalent to multiplying the polynomial by -1.

To Subtract Polynomials: Find the opposite of the polynomial being subtracted. Combine *like* terms.

For exercises 10 and 11, find the opposite of each polynomial.

10. $3r-15$

Opposite is $-(3r-15)$

$\boxed{-3r+15}$

11. $-4x^3+3x^2-6$

opposite is

$-(-4x^3+3x^2-6)$

$=-1(-4x^3+3x^2-6)$

$=\boxed{4x^3-3x^2+6}$

For exercises 12 – 16, subtract the polynomials.

12. $(6a^2-5)-(12a+5)$

$6a^2-5-1(12a+5)$

$6a^2-5-12a-5$

$\boxed{6a^2-12a-10}$

13. $5x^2y-(-x^2y+2xy+3)$

$5x^2-1(-x^2y+2xy+3)$

$5x^2y+x^2y-2xy-3$

$=\boxed{6x^2y-2xy-3}$

14. $\left(\frac{1}{4}g^2+\frac{2}{5}g-\frac{3}{7}\right)-\left(\frac{3}{4}g^2-\frac{1}{5}g-\frac{2}{7}\right)$

$\frac{1}{4}g^2+\frac{2}{5}g-\frac{3}{7}-\frac{3}{4}g^2+\frac{1}{5}g+\frac{2}{7}$

Put like terms together:

$\underbrace{\frac{1}{4}g^2-\frac{3}{4}g^2}+\underbrace{\frac{2}{5}g+\frac{1}{5}g}\underbrace{-\frac{3}{7}+\frac{2}{7}}$

already have common denominators!

$=-\frac{2}{4}g^2+\frac{3}{5}g-\frac{1}{7}$

$=\boxed{-\frac{1}{2}g^2+\frac{3}{5}g-\frac{1}{7}}$

15.

$$\begin{array}{rrrr} 1.2y^3 & -2.3y^2 & & +3.1 \\ -(2.4y^3 & & +4.8y & -2.1) \\ \hline \end{array}$$

$-1.2y^3-2.3y^3-4.8y+5.2$

16. $\left(12w^3+3w-6\right)-\left(-w^3-4w^2+3w+8\right)+\left(w^2-7w+1\right)$

$12w^3+3w-6+w^3+4w^2-3w-8+w^2-7w+1$

$\boxed{13w^3+5w^2-7w-13}$

17. Find the difference of $\left(-2t^2+3t-4\right)$ and $\left(-t^2-4t-4\right)$.

$(-2t^2+3t-4)-(-t^2-4t-4)$

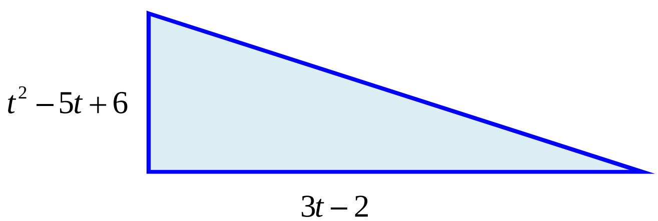
$=-2t^2+3t-4+t^2+4t+4$

$=-t^2+7t+0$

$=\boxed{-t^2+7t}$

Polynomials and Applications to Geometry

18. If the perimeter of the figure can be represented by the polynomial $3t^2-7t+1$, find a polynomial that represents the length of the missing side.



Omit the geometry