Multiplication of Polynomials and Special Products

Section 5.6

Multiplication of Polynomials

To multiply monomials: Use the associative and commutative properties of multiplication to group coefficients and like bases.

Simplify the result by using the properties of exponents.

For exercises 1 and 2, multiply the expressions.

1. $(b^4)(5b^5)$

$= \boxed{5b^9}$

2. $(4a^2b)(-a^4b^7)$

$\boxed{-4a^6b^8}$

To multiply polynomials: Use the distributive property: $a(b+c) = ab + ac$ also $(a+b)c = ac + bc$

Combine like terms.

For exercises 3 – 9, multiply the polynomials.

3. $4xy(2xy - 3x + 6y)$

$= 4xy(2xy) + 4xy(-3x) + 4xy(6y)$

$= \boxed{8x^2y^2 - 12x^2y + 24xy^2}$

4. $(-3j^2 + 4j - 2)(-5j)$

or $-5j(-3j^2 + 4j - 2)$

$= \boxed{15j^3 - 20j^2 + 10j}$

5. $(b-4)(b+6)$

$= b(b+6) - 4(b+6)$

$= b^2 + 6b - 4b - 24$

$= \boxed{b^2 + 2b - 24}$

6. $(4p-3)(p-2)$

$(4p-3)(p-2)$

$= 4p(p-2) - 3(p-2)$

$= 4p^2 - 8p - 3p + 6$

$= \boxed{4p^2 - 11p + 6}$

7. $(5a-4b)(2a+3b)$

$5a(2a+3b) - 4b(2a+3b)$

$= 10a^2 + 15ab - 8ab - 12b^2$

$= \boxed{10a^2 + 7ab - 12b^2}$

Ex $7\frac{1}{2}$ $(x^2 - 4x)(2x^3 + 8x)$

$= x^2(2x^3 + 8x) - 4x(2x^3 + 8x)$

$= \boxed{2x^5 + 8x^3 - 8x^4 - 32x^2}$

8. $(2a-3)(4a^2+6a+9)$

$(2a-3)(4a^2+6a+9)$

$= 2a(4a^2+6a+9) - 3(4a^2+6a+9)$

$= 8a^3 + \cancel{12a^2} + \cancel{18a} - \cancel{12a^2} - \cancel{18a} - 27$

$= \boxed{8a^3 - 27}$

Ex $8\frac{1}{2}$) $(x^2-7x-1)(2x^2-5x-6)$

$= x^2(2x^2-5x-6) - 7x(2x^2-5x-6)$
$- 1(2x^2-5x-6$

$= 2x^4 - 5x^3 - 6x^2$
$-14x^3 + 35x^2 + 42x$
$-2x^2 + 5x + 6$

$= \boxed{2x^4 - 19x^3 + 27x^2 + 47x + 6}$

9. ~~$5x^2-3x+4$ × $2x-1$~~

$(2x-1)(5x^2-3x+4)$

$= 2x(5x^2-3x+4) - 1(5x^2-3x+4)$

$= 10x^3 - 6x^2 + 8x$
$-5x^2 + 3x - 4$

$= \boxed{10x^3 - 11x^2 + 11x - 4}$

Special Case Products: Difference of Squares and Perfect Square Trinomials

The sum and difference of the same two terms are called conjugates.

For example, $2x-7$ and $2x+7$ are conjugates
$4+5x$ and $4-5x$ are conjugates

First special case: When you multiply conjugates your answer is the difference between the square of the first term and the square of the second term.

$$(a+b)(a-b)=a^2-b^2$$

The product is called a *difference of squares*.

$(a+b)(a-b)$
$= a(a-b) + b(a-b)$
$= a^2 - \cancel{ab} + \cancel{ba} - b^2$
$= a^2 - b^2$

For exercises 10 – 13, multiply the conjugates.

10. $(3x+5y)(3x-5y)$

$= 3x(3x-5y) + 5y(3x-5y)$

$= 9x^2 - \cancel{15xy} + \cancel{15yx} - 25y^2$

$= \boxed{9x^2 - 25y^2}$

In one step:
$(3x+5y)(3x-5y)$
$= (3x)^2 - (5y)^2 \leftarrow (a+b)(a-b) = a^2 - b^2$
$= \boxed{9x^2 - 25y^2}$

11. $(5-7a)(5+7a)$

In one step:

$(r)^2 - \left(\frac{2}{3}\right)^2$

$= \boxed{r^2 - \frac{4}{9}}$

12. $\left(r+\frac{2}{3}\right)\left(r-\frac{2}{3}\right)$

$r\left(r-\frac{2}{3}\right)+\frac{2}{3}\left(r-\frac{2}{3}\right)$

$= r^2 - \cancel{\frac{2}{3}r} + \cancel{\frac{2}{3}r} + \frac{2}{3}\left(-\frac{2}{3}\right)$

$= \boxed{r^2 - \frac{4}{9}}$

13. $\left(2-5x^2\right)\left(2+5x^2\right)$

Second special case: Squaring a binomial

For example: $(2x-5)^2$ or $(4x+7y)^2$

When squaring a binomial, the product will be a trinomial called a *perfect square trinomial*.

To square a binomial: The first and third terms are formed by squaring each term of the binomial. The middle term equals twice the product of the terms in the binomial.

$(a+b)(a+b)$

$= a(a+b)+b(a+b)$

$= a^2+ab+ba+b^2$

$= a^2+2ab+b^2$

$$(a+b)^2 = a^2+2ab+b^2 \qquad (a-b)^2 = a^2-2ab+b^2$$

For exercises 14 – 16, square the binomials.

14. $(c-5)^2$

$(c-5)(c-5)$

$= c(c-5)-5(c-5)$

$= c^2-5c-5c+25$

$= \boxed{c^2-10c+25}$

15. $(5d+9)^2$

16. $(3-5g)^2$

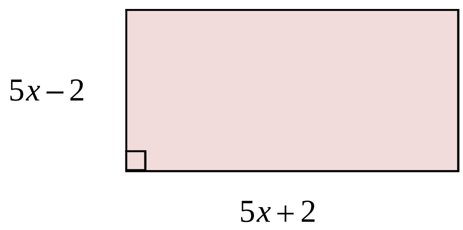
17. a. Evaluate $(5-3)^2$ by working within the parentheses first.

b. Evaluate 5^2-3^2.

c. Compare the answers to parts (a) and (b) and make a conjecture about $(a-b)^2$ and a^2-b^2.

Applications to Geometry

18. Find a polynomial expression that represents the area of the rectangle shown in the figure.



19. Find a polynomial that represents the volume of the rectangular solid shown in the figure. (Recall: $V = lwh$)