Division by a Monomial

Dividing a Polynomial by a Monomial If a, b, and c are polynomials such that $c \neq 0$, then

Ex: $\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad \text{Similarly,} \quad \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

Note: To divide a polynomial by a monomial, divide each individual term in the polynomial by the divisor and simplify the result.

For exercises 1 – 4, divide the polynomials .

1. $(4x^4 + 8x^2 - 2) \div (-2)$

$\frac{4x^4 + 8x^2 - 2}{-2} = \frac{4x^4}{-2} + \frac{8x^2}{-2} - \frac{2}{-2}$

$= \boxed{-2x^4 - 4x^2 + 1}$

2. $\dfrac{7k^4 - 5k^3 + 4k^2}{k^2} = \dfrac{7k^4 - 5k^3 + 4k^2}{k^2}$

$\frac{7k^4}{k^2} - \frac{5k^3}{k^2} + \frac{4k^2}{k^2}$

$= \frac{7k^2}{1} - \frac{5k}{1} + \frac{4}{1}$

$= \boxed{7k^2 - 5k + 4}$

3. $(25y^2 - 10y + 5) \div (5y^2)$

4. $\dfrac{-x^3y - x^2y^2 - xy^3}{-x^2y^2}$

$= \frac{-x^3y - x^2y^2 - xy^3}{-x^2y^2}$

$= \frac{-x^3y}{-x^2y^2} - \frac{x^2y^2}{-x^2y^2} - \frac{xy^3}{-x^2y^2}$

$= \frac{x}{y} + \frac{1}{1} + \frac{y}{x}$

$= \boxed{\frac{x}{y} + 1 + \frac{y}{x}}$

Long Division: when the denominator has 2 or more terms

Ex: Divide $\frac{379}{12}$

$$\begin{array}{r} 31 \\ 12\,\overline{)\,379} \\ -36 \\ \hline 19 \\ 12 \\ \hline 7 \end{array}$$

Quotient

Divisor $\overline{)\text{ Dividend}}$

Remainder

$$\frac{379}{12} = \boxed{31 + \frac{7}{12}}$$

Ex: Divide. $\frac{3x^2+7x+9}{x+2}$

$$\begin{array}{r} 3x \quad +1 \\ x+2\,\overline{)\,3x^2 + 7x + 9} \\ -(3x^2 + 6x) \\ \hline x + 9 \\ -(x + 2) \\ \hline 7 \end{array}$$

$x(?) = 3x^2$
answer: $3x$

$$\frac{3x^2+7x+9}{x+2} = \boxed{3x+1+\frac{7}{x+2}}$$

Ex: Divide. $\dfrac{x^3-6x^2+x+14}{x-5}$

$$\frac{x^3}{x} \qquad \frac{-x^2}{x}$$

$$\begin{array}{r|l} & x^2 - x - 4 \\ \hline x-5 & x^3 - 6x^2 + x + 14 \\ & -(x^3 - 5x^2) \qquad \leftarrow x^2(x-5) \\ \hline & -x^2 + x \\ & -(-x^2 + 5x) \qquad \leftarrow -x(x-5) \\ \hline & -4x + 14 \\ & -(-4x + 20) \\ \hline & -6 \end{array}$$

$$\frac{x^3-6x^2+x+14}{x-5} = \boxed{x^2 - x - 4 + \frac{-6}{x-5}}$$

$$= \boxed{x^2 - x - 4 - \frac{6}{x-5}}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\begin{array}{r|l} & \text{Quotient} \\ \hline \text{Divisor} & \text{Dividend} \\ & \vdots \\ & \text{Remainder} \end{array}$$

Multiply both sides by Divisor:

$$\underset{\text{(numerator)}}{\text{Dividend}} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$$

Check previous example by multiplying:

$$(x-5)(x^2-x-4)-6$$
$$= x^3 - x^2 - 4x$$
$$\quad - 5x^2 + 5x + 20 - 6$$
$$= x^3 - 6x^2 + x + 14 \qquad \checkmark \text{ok}$$

Long Division

If the divisor has two or more terms, a *long division* process similar to the division of real numbers is used.

The solution to a long division problem is usually written in the form: $\text{quotient} + \dfrac{\text{remainder}}{\text{divisior}}$

> **TIP:** Recall that
>
> - Long division is used when the divisor has two or more terms.
> - If the divisor has one term, then divide each term in the dividend by the monomial divisor.

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5. **a.** Divide $\left(c^2+6c-18\right)\div\left(c+8\right)$

$$\begin{array}{r} c-2 \\ c+8\,\overline{)\,c^2+6c-18} \\ \oplus\,\ominus\;-(c^2+8c) \\ \hline -2c-18 \\ \oplus\,\oplus\;-(-2c-16) \\ \hline -2 \end{array}$$

$\leftarrow c(c+8)$

Write our answer:

$$\frac{c^2+6c-18}{c+8} = \boxed{c-2+\frac{-2}{c+8}}$$

$$= \boxed{c-2-\frac{2}{c+8}}$$

b. Check by multiplying the quotient by the divisor and adding the remainder.

Dividend = (Quotient)(Divisor) + Remainder

$(c-2)(c+8)-2$

$= c(c+8)-2(c+8)-2$

$= c^2+8c-2c-16-2 = c^2+6c-18 \checkmark$ ok!

For exercises 6 – 10, divide the polynomials.

6. $\dfrac{y^2-2y+3}{y+1} = \boxed{y-3+\dfrac{6}{y+1}}$

$$\begin{array}{r} y-3 \\ y+1\,\overline{)\,y^2-2y+3} \\ \oplus\,\ominus\,\ominus\;-(y^2+y) \\ \hline -3y+3 \\ \oplus\,\oplus\,\oplus\;-(-3y-3) \\ \hline 6 \end{array}$$

Check: $(y+1)(y-3)+6$

$= y^2-3y+y-3+6 = y^2-2y+3 \checkmark$

7. $\left(6w^2+w-2\right)\div\left(3w+2\right)$

$$\begin{array}{r} 2w-1 \\ 3w+2\,\overline{)\,6w^2+w-2} \\ \oplus\,\ominus\,\ominus\;-(6w^2+4w) \\ \hline -3w-2 \\ \oplus\,\oplus\,\oplus\;-(-3w-2) \\ \hline 0 \end{array}$$

$$\frac{6w^2+w-2}{3w+2} = \boxed{2w-1}$$

8. $\left(2x^3+3x^2-2x+5\right)\div\left(2x+3\right)$

$$
\begin{array}{r}
x^2 + 0x - 1 \\
2x+3\,\overline{)\,2x^3+3x^2-2x+5} \\
-(2x^3+3x^2) \\
\hline
0x^2-2x \\
-(0x^2+0x) \\
\hline
-2x+5 \\
-(-2x-3) \\
\hline
8
\end{array}
$$

$$\frac{2x^3+3x^2-2x+5}{2x+3} = \boxed{x^2-1+\frac{8}{2x+3}}$$

Check it!

9. $\dfrac{6m^3-m^2-3}{3m-5} = \boxed{2m^2+3m+5+\dfrac{22}{3m-5}}$

$$
\begin{array}{r}
2m^2 + 3m + 5 \\
3m-5\,\overline{)\,6m^3-m^2+0m-3} \\
-(6m^3-10m^2) \\
\hline
9m^2+0m \\
-(9m^2-15m) \\
\hline
15m-3 \\
-(15m-25) \\
\hline
22
\end{array}
$$

10. $\left(2y^2+y^4-29\right)\div\left(y^2-5\right)$

$$
\begin{array}{r}
y^2 + 7 \\
y^2+0y-5\,\overline{)\,y^4+0y^3+2y^2+0y-29} \\
-(y^4+0y^3-5y^2) \\
\hline
7y^2+0y-29 \\
-(7y^2+0y-35) \\
\hline
6
\end{array}
$$

$$\frac{y^4+2y^2-29}{y^2-5} = \boxed{y^2+7+\frac{6}{y^2-5}}$$

Check: $(y^2+7)(y^2-5)+6$

$= y^4-5y^2+7y^2-35+6 = y^4+2y^2-29$ ✓ ok

11. There are two methods for dividing polynomials. When is long division necessary?

Example: Divide, $\dfrac{8x^3 - 10x^2 + x - 2}{2x-1}$

$$
\begin{array}{r|l}
 & 4x^2 - 3x - 1 \\
\hline
2x-1 & 8x^3 - 10x^2 + x - 2 \\
 & -(8x^3 - 4x^2) \\
\hline
 & -6x^2 + x \\
 & -(-6x^2 + 3x) \\
\hline
 & -2x - 2 \\
 & -(-2x + 1) \\
\hline
 & -3
\end{array}
$$

$$\frac{8x^3 - 10x^2 + x - 2}{2x - 1} = \boxed{4x^2 - 3x - 1 + \frac{-3}{2x-1}}$$

$$= \boxed{4x^2 - 3x - 1 - \frac{3}{2x-1}}$$

Check: $(2x-1)(4x^2 - 3x - 1) - 3$

$= 8x^3 - 6x^2 - 2x$
$\quad - 4x^2 + 3x + 1 - 3$

$= 8x^3 - 10x^2 + x - 2$ ✓ok

For exercises 12 – 15, determine which method to use to divide the polynomials: monomial division or long division. Then use that method to divide the polynomials.

12. $\dfrac{4a^5 - 8a^3 + 2a}{2a}$

13. $\left(8n^3 - 27\right) \div \left(2n - 3\right)$

14. $\left(h^4 + 2h^2 - 5\right) \div \left(h^2 + 3\right)$

15. $\left(6r^3 - 12r^2 + 9r - 3\right) \div \left(-3r^2\right)$