Identifying the Greatest Common Factor

Factor an integer: to write the integer as a product of two or more integers. $15 = 5 \cdot 3$ ← factored form of 15

Factor a polynomial: to express the polynomial as a product of two or more polynomials.

Ex: $x^2+7x+12 = (x+3)(x+4)$

Prime factorization of an integer: to write the integer as a product of prime numbers. Ex: 24 → $6 \cdot 4$ → $2 \cdot 3 \cdot 2 \cdot 2$; $2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3$

Greatest common factor (GCF) of two or more expressions: the product of the common factors, where each factor is raised to the lowest power to which it occurs in all the original expressions.

For exercises 1 – 4, identify the greatest common factor.

1. 12, 90

GCF = 6

2. $24a^3b$, $36a^4b$

GCF = $12a^3b$

To get the GCF of the variable part:
* if a variable shows up in all the expressions, then choose the smallest power
* if a variable is not in all the expressions, then the variable is not in the GCF.

3. $27x^2yz^2$, $6x^3yz$, $24yz^4$

GCF = $3yz$

4. $8(2x-3)$, $(2x-3)$

Factoring out the Greatest Common Factor

The process of factoring out a GCF is the reverse process of multiplying polynomials.

Both operations use the distributive property: $ab+ac=a(b+c)$

Factoring out the Greatest Common Factor

Step 1 Identify the GCF of all terms of the polynomial.

Step 2 Write each term as the product of the GCF and another factor.

Step 3 Use the distributive property to ~~remove~~ factor out the GCF.

Note: To check the factorization, multiply the polynomials to remove parentheses.

---

5. a. Use the distributive property to multiply $4(3x-7)$.

b. Use the distributive property to factor $12x-28$. GCF: 4

$4(3x-7)$

For exercises 6 – 10, factor out the GCF.

6. $6x-14$ GCF: 2

$6x-14$

$=2\left(\frac{6x}{2}-\frac{14}{2}\right)$

$=2(3x-7)$

7. $14f^3+21f^2+28f$ GCF: $7f$

$=7f\left(\frac{14f^3}{7f}+\frac{21f^2}{7f}+\frac{28f}{7f}\right)$

$=7f(2f^2+3f+4)$

Check: $7f(2f^2+3f+4)$

$=14f^3+21f^2+28f$ ✓

8. $20m^3n^4+5m^2n^2$ GCF: $5m^2n^2$

$20m^3n^4+5m^2n^2$

$=5m^2n^2(4mn^2+1)$

Check: $5m^2n^2(4mn^2+1)$

$=20m^3n^4+5m^2n^2$ ✓

9. a^3+2bc

a^3+2bc

GCF: 1

Could write:

$1(a^3+2bc)$

(not simplified)

This polynomial a^3+2bc is **prime** (cannot be factored except as a product of 1 and itself)

10. $8a^2b+a^3b^4-6a^3b^2+a^4b^3$

$8a^2b+a^3b^4-6a^3b^2+a^4b^3$

$a^2b(8+ab^3-6ab+a^2b^2)$

Factoring out a Negative Factor

Usually it is advantageous to factor out the opposite of the GCF when the leading coefficient of a polynomial is negative.

Note: This changes the signs of the remaining terms inside the parentheses.

---

11. For the polynomial $-3g^4-6g^2+9g$ GCF: $3g$

a. Factor out $-3g$.

$-3g(g^3+2g-3)$

Check: $-3g^4-6g^2+9g$ ✓

b. Factor out $3g$.

$3g(-g^3-2g+3)$

Check: $-3g^4-6g^2+9g$ ✓

For exercises 12 and 13, factor out the opposite of the greatest common factor.

12. $-b^3-2b^2+5b$

GCF: b

$-b(b^2+2b-5)$

Check: $-b^3-2b^2+5b$ ✓

13. $-2u+5v-w$

GCF: 1

$-1(2u-5v+w)$

Factoring out a Binomial Factor

For exercises 14 and 15, factor out the GCF.

14. $7(x^2+1)-6y(x^2+1)$

$(x^2+1)(7-6y)$

similar to $7u-6yu$

$u(7-6y)$

15. $3c(w^2-5)+(w^2-5)$

$(w^2-5)(3c+1)$

could write original as $3c(w^2-5)+1(w^2-5)$

similar to $3cx+x$

$x(3c+1)$

Factoring by Grouping

Factoring by Grouping To factor a four-term polynomial by grouping:

Step 1 Identify and factor out the GCF from all four terms.

step 1½: rewrite as a sum of parentheses ( ) + ( )

Step 2 Factor out the GCF from the first pair of terms.
Factor out the GCF from the second pair of terms.
(Sometimes it is necessary to factor out the opposite of the GCF.)

Step 3 If the two terms share a common binomial factor, factor out the binomial factor.

For exercises 16 – 19, factor by grouping.

16. $3a-6b+2a^2-4ab$

$(3a-6b)+(2a^2-4ab)$

$=3(a-2b)+2a(a-2b)$

$=(a-2b)(3+2a)$

Check: $(a-2b)(3+2a)$

$=3a+2a^2-6b-4ab$

compare to original ✓

17. $6x^2-4x-15x+10$

$(6x^2-4x)+(-15x+10)$

$=2x(3x-2)-5(3x-2)$

$=(3x-2)(2x-5)$

Check it:

$(3x-2)(2x-5)$

$6x^2-15x-4x+10$ ✓

Also correct: $(2x-5)(3x-2)$

GCF = 1

18. $18a^3b - 9a^2b^2 - 2a + b$

$(18a^3b - 9a^2b^2) + (-2a + b)$

$= 9a^2b(2a - b) - 1(2a - b)$

$= (2a - b)(9a^2b - 1)$

GCF: 3

19. $6x^2y + 6x^2 + 9y^3 + 9y^2$

$3(2x^2y + 2x^2 + 3y^3 + 3y^2)$

$= 3[(2x^2y + 2x^2) + (3y^3 + 3y^2)]$

$= 3[2x^2(y + 1) + 3y^2(y + 1)]$

$= 3[(y + 1)(2x^2 + 3y^2)]$

$= 3(y + 1)(2x^2 + 3y^2)$

19. Factor out $\frac{1}{3}$ from $\frac{4}{3}h^2 - \frac{2}{3}h + \frac{1}{3}$.

20. The formula $S = 2lw + 2hw + 2hl$ represents the surface area, S, of a rectangular prism with length, *l*, width, *w*, and height, *h*. Factor out the GCF and write an equivalent formula in factored form.

Prime number: a natural number larger than 1 that can only be factored as a product of 1 and itself (considering only positive integer factors)

Examples of prime numbers: 17, 2, 47, 3, 5, 19, 7, 11, 13

23

Numbers that are not prime (these are called composite numbers):

4, 6, 8, 122, 6, 9, 15 27, 16, ...

Note: 1 is neither prime nor composite.
(It is the multiplicative identity and is in a special category all by itself)

A **prime polynomial** is a polynomial that can only be factored (using integer coefficients) as a product of ± 1 and itself.