6.3: Factoring trinomials with leading coefficient 1.

Note Title 10/13/2016

Ex: Factor.

$$x^2 + 6x + 8$$

$$(x + 2)(x + 4)$$ ← answer

Check: $x^2 + 4x + 2x + 8$

$= x^2 + 6x + 8$ ✓

Note:

$(x+3)(x+4)$

$= x^2 + 4x + 3x + 12$

Factor.

Ex: $x^2 + 10x - 24$ (−) signs are opposite

$$(x - 2)(x + 12)$$

Check: $x^2 + 12x - 2x - 24$

$= x^2 + 10x - 24$

Because signs are opposite, I want a difference of 10

24
1·24
2·12
3·8
4·6

Ex: $x^2 + 10x + 24$ (+) same signs

$$(x + 4)(x + 6)$$

Check: $x^2 + 6x + 4x + 24$

$= x^2 + 10x + 24$

signs the same, so I want a sum of 10

24
1·24
2·12
3·8
4·6

Ex: $x^2 - 10x - 24$ (−) signs are opposite

$$(x + 2)(x - 12)$$

Check: $x^2 - 12x + 2x - 24$

$= x^2 - 10x - 24$ ✓

signs opp. so want a difference of 10

24
1·24
2·12
3·8
4·6

Ex: $x^2 - 10x + 24$ signs are the same (+)

$$(x - 4)(x - 6)$$

Check: $x^2 - 6x - 4x + 24$

$= x^2 - 10x + 24$

signs the same, so I want a sum of 10

24
1·24
2·12
3·8
4·6

Factoring Trinomials with a Leading Coefficient of 1

To factor x^2+bx+c as a product of two binomials:

Construct binomials of the form $(x+\ \ )(x+\ \ )$

The remaining terms can be obtained from two integers whose product is c and whose sum is b.

Note: If no such integers exist, then the trinomial is not factorable and is called a **prime polynomial**.

Sign Rules for Factoring Trinomials Given the trinomial x^2+bx+c, the signs within the binomial factors are determined as follows:

Case 1 If c is *positive*, then the signs in the binomials must be the same (either both positive or both negative). The correct choice is determined by the middle term. If the middle term is positive, then both signs must be positive. If the middle term is negative, then both signs must be negative.

c is positive, so want a sum of 3x for middle term (sum of 3)

$$x^2+3x+2$$
$$(x+1)(x+2)$$

Same signs

Check: $x^2+2x+1x+2 = x^2+3x+2$

c is positive.

$$x^2-3x+2$$
$$(x-1)(x-2)$$

Same signs

Note: This all assumes the leading term is positive

Case 2 If c is *negative*, then the signs in the binomials must be different.

c is negative., so want a difference of 4x for middle term (want a difference of 4)

$$x^2+4x-12$$
$$(x+6)(x-2)$$

Different signs

Check: $x^2-2x+6x-12$, $x^2+4x-12$

c is negative.

$$x^2-4x-12$$
$$(x-6)(x+2)$$

Different signs

For exercises 1 – 6, factor completely.

1. $x^2+15x+56$

(+) (+) same signs GCF: 1

$(x+7)(x+8)$

Check: $x^2+8x+7x+56 = x^2+15x+56$

same signs, so want sum of 15

56
1.56
2.28
4.14
7.8

2. $x^2-12x+32$

$(x-4)(x-8)$

$(x-8)(x-4)$

both correct

3. $y^2+3y-40$

4. c^2+6c-7

5. $t^2+14y+49$

Prime (doesn't factor)

5b $t^2+14t+49$

$(t+7)(t+7)$

$=(t+7)^2$

6. d^2-3d+4

(+) same signs sum of 3

4

1·4

2·2

none of these pairs have a sum of 3, so it is

Prime

For exercises 7 and 8, assume that b and c represent positive integers.

7. When factoring a polynomial of the form x^2-bx+c, pick an appropriate combination of signs.

a. $(x+\ \ )(x+\ \ )$ b. $(x+\ \ )(x-\ \ )$ c. $(x-\ \ )(x-\ \ )$

8. When factoring a polynomial of the form x^2+bx-c, pick an appropriate combination of signs.

a. $(x+\ \ )(x+\ \ )$ b. $(x+\ \ )(x-\ \ )$ c. $(x-\ \ )(x-\ \ )$

Important guidelines:

- To factor a trinomial, write the trinomial in descending order such as x^2+bx+c.
- For all factoring problems, *always* factor out the GCF from all terms first.

For exercises 9 – 16, factor completely.

9. $8x+x^2-20$

(-) signs opposite want a difference of 8

$x^2+8x-20$

$(x-2)(x+10)$

Check: $x^2+10x-2x-20$

$=x^2+8x-20$ ✓

20

1·20

2·10

4·5

10. $3b^2-9b+6$

(+) signs are the same want a sum of 3

$3(b^2-3b+2)$

$3(b-1)(b-2)$

2

1·2

Check: $(b-1)(b-2)$

$=b^2-2b-1b+2$

$=b^2-3b+2$ ✓

11. $4x^3 - 8x^2 - 60x$

(−) signs are opposite

$4x(x^2 - 2x - 15)$

$4x(x+3)(x-5)$

check: $4x^3 - 8x^2 - 60x$ ✓

check: $(x+3)(x-5)$

$= x^2 - 5x + 3x - 15$

$= x^2 - 2x - 15$ ✓

signs are opposite, so want a difference of 2

15 → 1·15 → 3·5

12. $-m^2 - 10m - 16$

Factor out −1:

(+) same signs want a sum of 10

$-1(m^2 + 10m + 16)$

$-1(m+2)(m+8)$

usually write $-(m+2)(m+8)$

Check: $(m+2)(m+8)$

$= m^2 + 8m + 2m + 16$

16 → 1·16 → 2·8 → 4·4

13. $10c - 2c^2 + 12$

$-2c^2 + 10c + 12$

(−) signs are opposite want a difference of 5

$-2(c^2 - 5c - 6)$

$-2(c+1)(c-6)$

check: $c^2 - 6c + 1c - 6$

$= c^2 - 5c - 6$

6 → 1·6 → 2·3

14. $s^2 - 11st + 24t^2$ $= m^2 + 10m + 16$ ✓

$s^2 - 11st + 24t^2$

(+) same signs want sum of 11

$(s - 3t)(s - 8t)$

Check:

$s^2 - 8st - 3st + 24t^2$

$= s^2 - 11st + 24t^2$ ✓

24 → 1·24 → 2·12 → 3·8 → 4·6

15. $p^4 + 4p^2 - 45$

$(p^2 - 5)(p^2 + 9)$

Check

$p^4 + 9p^2 - 5p^2 - 45$

$= p^4 + 4p^2 - 45$ ✓

45 → 1·45 → 3·15 → 5·9

16. $a^3b^2 + 14a^2b^2 + 48ab^2$

17. A student factored a trinomial as $(3x+9)(x-4)$. The instructor did not give full credit. Why?

$(3x+9)(x-4)$ → $3x^2 - 3x - 36$

Factor out 3:

$3(x+3)(x-4)$

OR take GCF out 1st (recommend)

$3x^2 - 3x - 36$

(−) difference of 1

$= 3(x^2 - x - 12)$

$= 3(x-4)(x+3)$

18. What polynomial factors as $(x+5)(x-12)$?