6.3: Factoring trinomials ax^2+bx+c (when the leading coefficient > 1)

Note Title 10/18/2016

Ex: Factor.

(+) same signs

$$2x^2+9x+10$$

Try $(2x+1)(x+10)$

Check: $2x^2+20x+1x+10$

$=2x^2+21x+10$ No!

10
1 · 10
2 · 5

Try $(2x+10)(x-1)$

$2x^2-2x+10x-10$

$2x^2+8x-10$ No!

Want $2x^2+9x+10$

Try $(2x+10)(x+1)$

$2x^2+2x+10x+10$

$2x^2+12x+10$ No!

Try $(2x+2)(x+5)$

$2x^2+10x+2x+10$

$2x^2+12x+10$

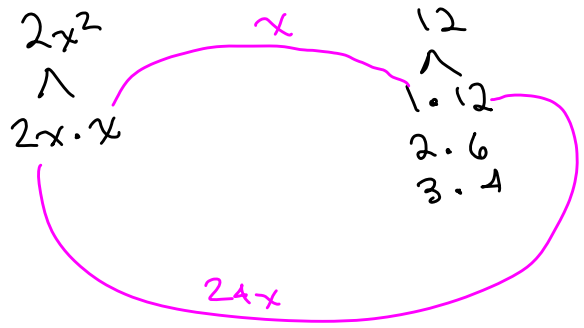
Try $\boxed{(2x+5)(x+2)}$

$2x^2+4x+5x+10$

$=2x^2+9x+10$ ✓

Ex: $2x^2 - 25x + 12$ (+) same signs want a sum of 25x for middle term

$(2x - 1)(x - 12)$



Check:

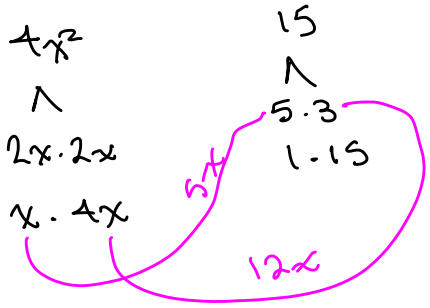
$2x^2 - 24x - 1x + 12$

$2x^2 - 25x + 12$

Example: Factor.

$4x^2 - 7x - 15$ (-) signs are opposite difference of 7x for middle term

$\boxed{(x - 3)(4x + 5)}$



Check:

$4x^2 + 5x - 12x - 15$

$= 4x^2 - 7x - 15$ ✓

Note: $\boxed{(4x + 5)(x - 3)}$ also correct

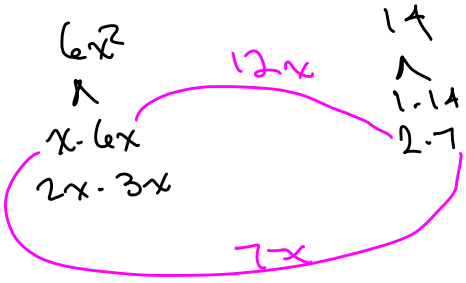
Check: $4x^2 - 12x + 5x - 15$

$= 4x^2 - 7x - 15$ ✓

Example: Factor

$6x^2 + 19x + 14$ (+) signs are the same so we want a sum of 19x for middle term

$(x + 2)(6x + 7)$



Check: $6x^2 + 7x + 12x + 14$

$= 6x^2 + 19x + 14$ ✓

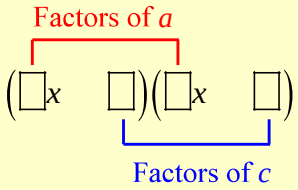
Factoring Trinomials by the Trial-and-Error Method

Trial-and-Error Method to Factor ax^2+bx+c

Step 1 Factor out the GCF.

Step 2 List all pairs of positive factors of a and pairs of positive factors of c. Consider the reverse order for one of the lists of factors.

Step 3 Construct two binomials of the form:



Step 4 Test each combination of factors and signs until the correct product is found.

Step 5 If no combination of factors produces the correct product, the trinomial cannot be factored further and is a **prime polynomial**.

Two important guidelines:

- For any factoring problem you encounter, always factor out the GCF from all terms first.
- To factor a trinomial, write the trinomial in the form ax^2+bx+c.

For exercises 1 and 2, assume a, b, and c represent positive integers.

1. When factoring a polynomial of the form ax^2+bx+c, pick an appropriate combination of signs.

a. $(\ +\ )(\ +\ )$ **b.** $(\ +\ )(\ -\ )$ **c.** $(\ -\ )(\ -\ )$

2. When factoring a polynomial of the form ax^2-bx-c, pick an appropriate combination of signs.

a. $(\ +\ )(\ +\ )$ **b.** $(\ +\ )(\ -\ )$ **c.** $(\ -\ )(\ -\ )$

For exercises 3 – 6, factor completely by using the trial-and-error method.

3. $2y^2 + y - 15$

(−) signs opposite want difference of 1y

$(2y - 5)(y + 3)$

$2y^2$ → $y \cdot 2y$; 15 → $1 \cdot 15$, $3 \cdot 5$; $-5y$

4. $3x^2 + 14x + 8$

(+) same signs want sum of 14x for middle term

$(3x + 2)(x + 4)$

$3x^2$ → $3x \cdot x$; 8 → $1 \cdot 8$, $2 \cdot 4$; $2x$, $12x$

5. $8t^2 - 18t + 9$

(+) same signs want sum of 18t

$(2t - 3)(4t - 3)$

$8t^2$ → $t \cdot 8t$, $2t \cdot 4t$; 9 → $1 \cdot 9$, $3 \cdot 3$; $-12t$, $-6t$

6. $18m + 5m^2 - 8$

opposite signs (−) difference of 18m

$5m^2 + 18m - 8$

$(5m - 2)(m + 4)$

$5m^2$ → $m \cdot 5m$; 8 → $1 \cdot 8$, $2 \cdot 4$; $2m$, $20m$

For exercises 7 and 8, be sure to factor out the GCF first.

7. $-a^2 + 20a - 36$

(+) same signs sum of 20a

$-1(a^2 - 20a + 36)$

$-(a - 2)(a - 18)$

Check: $(a-2)(a-18)$
$= a^2 - 18a - 2a + 36$
$= a^2 - 20a + 36$ ✓

8. $120w^2 + 20wy - 140y^2$

(−) opposite signs want a difference of 1wy

$20(6w^2 + wy - 7y^2)$

$20(w - y)(6w + 7y)$

$6w^2$ → $w \cdot 6w$, $2w \cdot 3w$; $7y^2$ → $7y \cdot y$; $-6wy$, $7wy$

Note: this is similar to $6w^2 + w - 7$

For exercises 9 and 10, factor the higher degree polynomial.

9. $6y^4+23y^2-18$

$6(y^2)^2+23y^2-18$

Let $u=y^2$:

$6u^2+23u-18$

$(2u+9)(3u-2)$

$6u^2$ | 18

$u\cdot 6u$ | $1\cdot 18$

$2u\cdot 3u$ | $2\cdot 9$

$3\cdot 6$

$4u$

$27u$

Put in y^2 for u:

$(2y^2+9)(3y^2-2)$

Check $6y^4-4y^2+27y^2-18$

$=6y^4+23y^2-18$

10. g^4+2g^2-48

$(g^2+8)(g^2-6)$

(−) opposite signs

want a difference of $2g^2$

g^4

$g^2\cdot g^2$

$6g^2$

$8g^2$

48

$1\cdot 48$

$2\cdot 24$

$3\cdot 16$

$4\cdot 12$

$6\cdot 8$

Check:

$g^4-6g^2+8g^2-48$

g^4+2g^2-48

For exercises 11 – 14, factor the trinomial completely.

11. $24+10z-4z^2$

12. $x^2+13xy+12y^2$

13. $a^2b^2+3ab^2-70b^2$

14. $10z^3+84z^2+32z$