Factoring a Difference of Squares

Recall that the product of two conjugates results in a **difference of squares**:

$$(a+b)(a-b)=a^2-b^2$$

$a^2-ab+ab-b^2$

$=a^2-b^2$

Therefore, to factor a difference of squares, the process is reversed.

Factored Form of a Difference of Squares	$a^2-b^2=(a+b)(a-b)$

Sum of Squares Suppose *a* and *b* have no common factor. Then the sum of squares a^2+b^2 is *not* factorable over the real numbers. That is, a^2+b^2 is *prime* over the real numbers.

Remember: For any factoring problem you encounter, always factor out the GCF from all terms first.

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1. What binomial factors as $(2x-3)(2x+3)$?

$a=2x$

$b=3$

$4x^2+6x-6x-9$

$=\boxed{4x^2-9}$

For exercises 2 – 7, factor the binomials completely.

2. y^2-49

y^2-7^2

$=\boxed{(y-7)(y+7)}$

Check: $y^2+7y-7y-49$

3. $9s^2-25t^2$

$(3s)^2-(5t)^2$

$\boxed{(3s+5t)(3s-5t)}$

4. $50-2d^2$

$-2d^2+50$

$-2(d^2-25)$

$\boxed{-2(d+5)(d-5)}$

Check: $(d+5)(d-5)$

$d^2-5d+5d-25$

d^2-25 ✓

5. $64x^2y^2-1$

$(8xy)^2-(1)^2$

$\boxed{(8xy+1)(8xy-1)}$

x^2 + positive is always prime

6. h^4-81

$(h^2)^2-(9)^2$

$(h^2+9)(h^2-9)$

$\boxed{(h^2+9)(h+3)(h-3)}$

7. $4x^2+9$

prime

Ex: Factor

$y^2 - 36$

$\boxed{(y+6)(y-6)}$

Check: $y^2 - \cancel{6y} + \cancel{6y} - 36$

$y^2 - 36$ ✓

Ex: Factor

$x^3 - 81xy^2$

$x(x^2 - 81y^2)$

$x(x^2 - (9y)^2)$

$= \boxed{x(x+9y)(x-9y)}$

Ex: Factor

$x^2 + 25$

$\boxed{\text{prime}}$

prime: doesn't factor

Why is $x^2 + 25$ prime?

Rewrite: $x^2 + 0x + 25$

(+) same signs want a sum of $0x$

25
$\wedge$
$1 \cdot 25$
$5 \cdot 5$

A sum of 0 is impossible

Note: $x^2 - 25$

$\boxed{(x+5)(x-5)}$

Rewrite: $x^2 + 0x - 25$

(−) signs are opposite want a difference of 0

25
$\wedge$
$1 \cdot 25$
$5 \cdot 5$

difference of 0 → $5 \cdot 5$

For exercises 8 and 9, factor the polynomials completely.

8. $x^3 - 3x^2 - 4x + 12$

$(x^3 - 3x^2) + (-4x + 12)$

$x^2(x-3) - 4(x-3)$

$(x-3)(x^2-4)$

$\boxed{(x-3)(x+2)(x-2)}$

9. $x^2y^2 - x^2 + y^4 - y^2$

$(x^2y^2 - x^2) + (y^4 - y^2)$

$x^2(y^2 - 1) + y^2(y^2 - 1)$

$(y^2-1)(x^2+y^2)$

$\boxed{(y+1)(y-1)(x^2+y^2)}$

Factoring Perfact Square Trinomials

Recall that the square of a binomial always results in a **perfect square trinomial.**

$$(a+b)^2 = (a+b)(a+b) \xrightarrow{\text{Multiply}} = a^2 + 2ab + b^2$$

$$(a-b)^2 = (a-b)(a-b) \xrightarrow{\text{Multiply}} = a^2 - 2ab + b^2$$

Factored Form of a Perfect Square Trinomial

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 - 2ab + b^2 = (a-b)^2$$

Checking for a Perfect Square Trinomial

Step 1 Determine whether the first and third terms are both perfect squares and have positive coefficients.

Step 2 If this is the case, identify a and b, and determine if the middle term equals $2ab$ or $-2ab$.

Ex.:

Factor

$x^2 + 6x + 9$

(+) same signs sum of 6

$(x+3)(x+3)$

9

1·9

3·3

Check

$x^2 + 3x + 3x + 9$

$= x^2 + 6x + 9$ ✓

Final answer:

$\boxed{(x+3)^2}$

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10. Multiply. $(3x-4)^2$

11. **a.** Which trinomial is a perfect square trinomial? $x^2 + 6x + 9$ or $x^2 + 10x + 9$

b. Factor the trinomials from part (a).

For exercises 12 – 17, factor completely. (*Hint*: Look for the pattern of a perfect square trinomial.)

12. $a^2 - 16a + 64$

$(a - 8)(a - 8)$

Check: $a^2 - 8a - 8a + 64$

$a^2 - 16a + 64$ ✓

$\boxed{(a-8)^2}$

64

$1 \cdot 64$

$2 \cdot 32$

$4 \cdot 16$

$8 \cdot 8$

13. $9x^2 + 30x + 25$

Try $\boxed{(3x + 5)^2}$

Check: $(3x+5)(3x+5)$

$9x^2 + 15x + 15x + 25$

$9x^2 + 30x + 25$ ✓

14. $50 + 2d^2 - 20d$

15. $9x^2 + 6xy + y^2$

16. $4x^2 + 25x + 25$

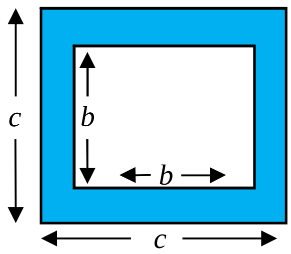
17. $12 + 27s^2 - 36s$

For exercises 18 and 19, factor the difference of squares.

18. $(3z+1)^2 - 81$

19. $25 - (x-3)^2$

20. **a.** Write a polynomial that represents the area of the shaded region in the figure.



b. Factor the expression from part (a).