Definition of a Quadratic Equation.

A linear equation in one variable is an equation of the form $ax+b=c \; (a \neq 0)$.

A linear equation in one variable is sometimes called a first-degree polynomial equation because the highest degree of all its terms is 1.

A second-degree polynomial equation in one variable is called a quadratic equation.

> **A Quadratic Equation in One Variable**
>
> If *a*, *b*, and *c* are real numbers such that $a \neq 0$, then a **quadratic equation** is an equation that can be written in the form
> $$ax^2+bx+c=0$$

standard form for a quadratic equation

For exercises 1 – 3, identify the equation as linear, quadratic, or neither.

1. $4x^2-16x=0$

quadratic

Note: $a=4$, $b=-16$, $c=0$

2. $5x-2=0$

linear

3. $12-x^4=0$

neither

Zero Product Rule

One method for solving a quadratic equation is to factor and apply the zero product rule.

(Zero product property of real numbers)

The zero product rule states that if the product of two factors is zero, then one or both of its factors is zero.

> **Zero Product Rule** If $ab=0$, then $a=0$ or $b=0$.

(both could possibly be 0)

Note: The zero product rule can be used to solve higher degree polynomial equations provided the equations can be set to zero and written in factored form.

For exercises 4 – 7, solve the equations using the zero product rule.

Note: I could rewrite this equation:
$8x^2+10x-4x-5=0$
$8x^2+6x-5=0$

4. $(x-3)(x+8)=0$

$x-3=0$ OR $x+8=0$
$+3 \quad +3$ $\quad -8 \quad -8$
$x=3$ $\quad x=-8$

$\boxed{\text{Solution Set: } \{3,-8\}}$

Check: $x=3$: $(3-3)(3+8)=0$
$0(11)=0$
$0=0$ True

$x=-8$: $(-8-3)(-8+8)=0$
$(-11)(0)=0$
$0=0$ ✓
True

5. $(2x-1)(4x+5)=0$

$2x-1=0$ OR $4x+5=0$
$2x=1$ $\quad 4x=-5$
$\frac{2x}{2}=\frac{1}{2}$ $\quad \frac{4x}{4}=\frac{-5}{4}$
$x=\frac{1}{2}$ $\quad x=-\frac{5}{4}$

Solution Set: $\boxed{\{\frac{1}{2},-\frac{5}{4}\}}$

6. $3(x-6)(3x+1)=0$

$3=0$ or $x-6=0$ or $3x+1=0$

will never be true | $x=6$ | $3x=-1$

$x=-\frac{1}{3}$

Solution set: $\boxed{\{6, -\frac{1}{3}\}}$

7. $x(5x-9)=0$

$x=0$ or $5x-9=0$

$5x=9$

$\frac{5x}{5}=\frac{9}{5}$

$x=\frac{9}{5}$

Sol'n Set: $\{0, \frac{9}{5}\}$

Solving Equations by Factoring

Solving a Quadratic Equation by Factoring

Step 1 Write the equation in the form: $ax^2+bx+c=0$
(standard form)

Step 2 Factor the quadratic expression completely.

Step 3 Apply the zero product rule. That is, set each factor equal to zero, and solve the resulting equations.

Note: The solution(s) found in step 3 may be checked by substitution in the original equation.

Important: The zero product rule can be used to solve higher degree polynomial equations provided the equations can be set to zero and written in factored form.

For exercises 8 – 21, solve the equations.

Check:

$b=5$: $5^2+3(5)-40=0$

$25+15-40=0$

$40-40=0$ ✓

$b=-8$: $(-8)^2+3(-8)-40=0$

$64-24-40=0$

$40-40=0$ ✓

8. $b^2+3b-40=0$

$(b-5)(b+8)=0$

$b-5=0$ or $b+8=0$

$b=5$ | $b=-8$

$\boxed{\text{Sol'n Set: } \{5, -8\}}$

Check:

$\underbrace{b^2+8b-5b}_{3b}-40$

40

1 · 40

2 · 20

4 · 10

5 · 8

9. $c^2-4c+3=0$

10. $4r^2-17r=15$
$-15 \quad -15$

$4r^2-17r-15=0$

$(4r+3)(r-5)=0$

$4r+3=0$ | $r-5=0$

$4r=-3$ | $r=5$

$\frac{4r}{4}=\frac{-3}{4}$

$r=-\frac{3}{4}$

Sol'n Set $\boxed{\{-\frac{3}{4}, 5\}}$

$4r^2$: $4r \cdot r$, $2r \cdot 2r$

15: $1 \cdot 15$, $3 \cdot 5$

$3r$

$20r$

Check

$(4r+3)(r-5)$

$4r^2-20r+3r-15$

$4r^2-17r-15$ ✓

11. $0=25x^2-36$

$\pm\frac{6}{5}$

"plus or minus $\frac{6}{5}$"

$25x^2-36=0$

$(5x)^2-(6)^2=0$

$(5x+6)(5x-6)=0$

$5x+6=0$ | $5x-6=0$

$5x=-6$ | $5x=6$

$x=-\frac{6}{5}$ | $x=\frac{6}{5}$

Sol'n Set: $\boxed{\{-\frac{6}{5}, \frac{6}{5}\}}$

OR $\boxed{\{\pm\frac{6}{5}\}}$

12. $0 = 3t^2 - 18t - 48$

$3t^2 - 18t - 48 = 0$

$3(t^2 - 6t - 16) = 0$

$3(t+2)(t-8) = 0$

$3 = 0$ never true no matter what t is | $t + 2 = 0$, $t = -2$ | $t - 8 = 0$, $t = 8$

16
1·16
2·8
4·4

Sol'n Set: $\boxed{\{-2, 8\}}$

13. $4n^3 + 8n^2 + 3n = 0$

$n(4n^2 + 8n + 3) = 0$

$n(2n+1)(2n+3) = 0$

$n = 0$ | $2n + 1 = 0$, $2n = -1$, $n = -\frac{1}{2}$ | $2n + 3 = 0$, $2n = -3$, $n = -\frac{3}{2}$

Sol'n Set: $\boxed{\{0, -\frac{1}{2}, -\frac{3}{2}\}}$

14. $x^3 - 81x = 0$

$x(x^2 - 81) = 0$

$x(x-9)(x+9) = 0$

$x = 0$ | $x - 9 = 0$, $x = 9$ | $x + 9 = 0$, $x = -9$

Sol'n Set: $\boxed{\{0, 9, -9\}}$

can write: $\boxed{\{0, \pm 9\}}$

15. $9m^2 = 4$

$9m^2 - 4 = 0$

$(3m-2)(3m+2) = 0$

(diff. of 2 squares)

$3m - 2 = 0$, $3m = 2$, $\frac{3m}{3} = \frac{2}{3}$, $m = \frac{2}{3}$ | $3m + 2 = 0$, $3m = -2$, $\frac{3m}{3} = \frac{-2}{3}$, $m = -\frac{2}{3}$

Sol'n Set: $\boxed{\{\frac{2}{3}, -\frac{2}{3}\}}$

or $\boxed{\{\pm \frac{2}{3}\}}$

16. $6x - (x-3) = 2(x+1)$

$6x - x + 3 = 2x + 2$

$5x + 3 = 2x + 2$

$-2x - 2 \quad -2x - 2$

$3x + 1 = 0$

$3x = -1$

$\frac{3x}{3} = \frac{-1}{3}$

$x = -\frac{1}{3}$

Sol'n Set: $\boxed{\{-\frac{1}{3}\}}$

17. $d(3d + 14) = 5$

$3d^2 + 14d = 5$

$3d^2 + 14d - 5 = 0$

$(3d-1)(d+5) = 0$

$3d - 1 = 0$, $3d = 1$, $d = \frac{1}{3}$ | $d + 5 = 0$, $d = -5$

Sol'n Set: $\boxed{\{\frac{1}{3}, -5\}}$

18. $2x^3 - 5x = 3x^2$

$$2x^3 - 3x^2 - 5x = 0$$

$$x(2x^2 - 3x - 5) = 0$$

$$x(2x - 5)(x + 1) = 0$$

$x = 0$ | $2x - 5 = 0$, $2x = 5$, $x = \frac{5}{2}$ | $x + 1 = 0$, $x = -1$

Sol'n Set: $\boxed{\{0, \frac{5}{2}, -1\}}$

19. $6(a^2 + 2) = 25a - 2$

$$6a^2 + 12 = 25a - 2$$

$-25a + 2 \quad -25a \quad +2$

$$6a^2 - 25a + 14 = 0$$

$$(2a - 7)(3a - 2) = 0$$

$6a^2$: $a \cdot 6a$, $2a \cdot 3a$ $\quad$ 14: $1 \cdot 14$, $2 \cdot 7$ $\quad$ $4a$ $\quad$ $21a$

$2a - 7 = 0$, $2a = 7$, $a = \frac{7}{2}$ | $3a - 2 = 0$, $3a = 2$, $a = \frac{2}{3}$

Sol'n Set: $\{\frac{7}{2}, \frac{2}{3}\}$

20. $3s^3 + s^2 - 12s - 4 = 0$

$$(3s^3 + s^2) + (-12s - 4) = 0$$

$$s^2(3s + 1) - 4(3s + 1) = 0$$

$$(3s + 1)(s^2 - 4) = 0$$

$$(3s + 1)(s + 2)(s - 2) = 0$$

$3s + 1 = 0$, $3s = -1$, $s = -\frac{1}{3}$ | $s + 2 = 0$, $s = -2$ | $s - 2 = 0$, $s = 2$

Sol'n Set: $\boxed{\{-\frac{1}{3}, -2, 2\}}$

OR $\{-\frac{1}{3}, \pm 2\}$

21. $(k + 5)(k + 7) = 3$

$$k^2 + 12k + 35 = 3$$

$$k^2 + 12k + 32 = 0$$

$$(k + 8)(k + 4) = 0$$

$k + 8 = 0$, $k = -8$ | $k + 4 = 0$, $k = -4$

Sol'n Set: $\boxed{\{-8, -4\}}$