Applications of Quadratic Equations

1. If five is added to three times the $\underbrace{\text{square of a number}}_{x^2}$, the result is $\underbrace{\text{sixteen times the number}}_{16x}$. Find all such numbers.

Let x = the unknown number

$$3x^2 + 5 = 16x$$
$$3x^2 - 16x + 5 = 0$$
$$(3x - 1)(x - 5) = 0$$
$$3x - 1 = 0 \quad | \quad x - 5 = 0$$
$$3x = 1 \quad | \quad x = 5$$
$$x = \frac{1}{3}$$

The numbers are $\frac{1}{3}$ and 5.

2. The sum of the squares of two consecutive odd integers is nineteen more than the product of the integers. Find all such integers.

1st odd integer: x
2nd odd integer: $x+2$

$$x^2 + (x+2)^2 = x(x+2) + 19$$
$$x^2 + (x+2)(x+2) = x^2 + 2x + 19$$
$$x^2 + x^2 + 2x + 2x + 4 = x^2 + 2x + 19$$
$$x^2 + \underbrace{x^2 + 4x + 4}_{(x+2)^2} = x^2 + 2x + 19$$
$$2x^2 + 4x + 4 = x^2 + 2x + 19$$
$$-x^2 \quad -2x \quad -19 \quad -x^2 \quad -2x \quad -19$$
$$x^2 + 2x - 15 = 0$$
$$(x - 3)(x + 5) = 0$$
$$x - 3 = 0 \quad | \quad x + 5 = 0$$
$$x = 3 \quad | \quad x = -5$$

$x = 3$: 1st odd integer: $x = 3$
2nd odd integer: $x+2 = 3+2 = 5$

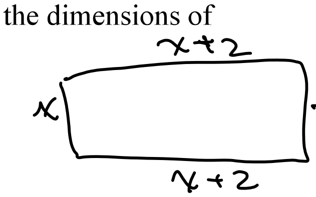
So one integer pair is 3, 5.

$x = -5$: 1st odd integer: $x = -5$
2nd odd integer: $x+2 = -5+2 = -3$

So another pair of integers is $-5, -3$.

3. The length of a rectangular garden is 2 ft more than the $\underbrace{\text{width}}_{x}$. If the area is 120 ft^2, what are the dimensions of the garden?

length: $x+2$
width: x



$$\text{Area} = (\text{length})(\text{width})$$
$$120 = (x+2)(x)$$
$$x(x+2) = 120$$
$$x^2 + 2x = 120$$
$$x^2 + 2x - 120 = 0$$
$$(x + 12)(x - 10) = 0$$
$$x + 12 = 0 \quad | \quad x - 10 = 0$$
$$x = -12 \quad | \quad x = 10$$

-12 does not make sense for width. Discard.

width: $x = 10$
length: $x+2$
$x = 10 \Rightarrow = 10 + 2 = 12$

The width is 10 ft and the length is 12 ft.

$\boxed{\text{6.8 cont'd}}$

4. The base of a triangle is twice the height. If the area is 36 in^2, find the base and the height.

base: $2x$
height: x

Area of Triangle $= \frac{1}{2}$ (base) (height)

$36 = \frac{1}{2}(2x)(x)$

$36 = \frac{2x^2}{2}$

$36 = x^2$

$0 = x^2 - 36$

$0 = (x+6)(x-6)$

$x + 6 = 0$
$x = -6$
Discard. Negative does not make sense for a dimension

$x - 6 = 0$
$x = 6$
only solution that work

$x = 6$

height: $x = 6$ in

base: $2x$

$x = 6" \Rightarrow 2(6\text{ in}) = 12$ in

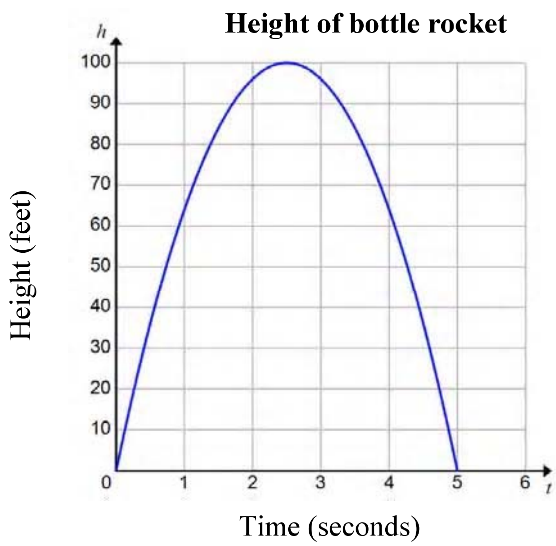
The base is ~~6 in~~ and the height is 12 in.

5. A bottle rocket is shot straight up into the air from the ground with an initial speed of 80 ft/sec. The height of the bottle rocket (in feet) is given by the equation

$h = -16t^2 + 80t$ where t is the time in seconds after the bottle rocket is shot into the air $(t \geq 0)$.

Find the time(s) when the bottle rocket is at ground level.

skip?



Example $4\frac{1}{2}$: The height of a triangle is 4 ft less than three times its base. The area is 42 ft^2. Find the base and height.

height $\xrightarrow[\text{to}]{\text{compare}}$ base

x

base: x

height: $3x - 4$

$$\text{Area of triangle} = \frac{1}{2}(\text{base})(\text{height})$$

$$42 = \frac{1}{2}(x)(3x-4)$$

$$\frac{1}{2}x(3x-4) = 42$$

$$\frac{1}{2}x(3x) + \frac{1}{2}x(-4) = 42$$

$$\frac{3x^2}{2} - \frac{4x}{2} = 42$$

Multiply both sides by 2 to clear the fractions.

$$(\cancel{2})\frac{3x^2}{\cancel{2}} - (2)2x = 42(2)$$

$$3x^2 - 4x = 84$$

$$3x^2 - 4x - 84 = 0$$

$$(3x + 14)(x - 6) = 0$$

$3x + 14 = 0$	$x - 6 = 0$
$3x = -14$	$x = 6$
$x = -\frac{14}{3}$	only value for x that works.
Discard	

$3x^2$ 84

$x \cdot 3x$ $1 \cdot 84$, $2 \cdot 42$, $3 \cdot 28$, $4 \cdot 21$, $6 \cdot 14$, $7 \cdot 12$

$18x$ $14x$

check:

$(3x+14)(x-6)$

$= 3x^2 - 18x + 14x - 84$

$= 3x^2 - 4x - 84$ ✓

$$\begin{array}{r} 14 \\ \times\ 6 \\ \hline 84 \end{array}$$

$\boxed{x = 6}$ base: $x = 6$

height: $3x - 4$

$x = 6 \Rightarrow$ $= 3(6) - 4$

$= 18 - 4$

$= 14$

$\boxed{\text{The base is 6 ft and the height is 14 ft.}}$

Check it! (against the words of the original problem)

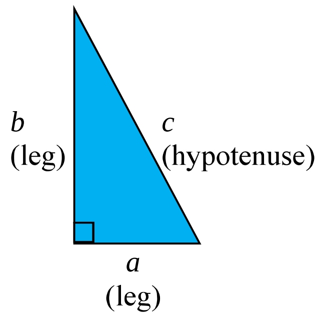
Pythagorean Theorem

Recall that a right triangle is a triangle that contains a 90° angle.

The Pythagorean theorem tells us that for a right triangle, the sum of the squares of the two legs equals the square of the hypotenuse.

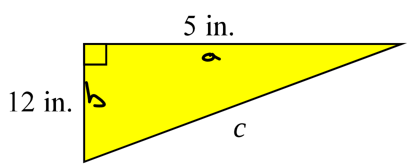
hypotenuse = side across from the 90° (right) angle

$$a^2 + b^2 = c^2$$



For exercises 6 and 7, find the length of the missing side of the right triangle.

6.



$a^2 + b^2 = c^2$

$5^2 + 12^2 = c^2$

$25 + 144 = c^2$

$169 = c^2$

$0 = c^2 - 169$

$0 = (c+13)(c-13)$

$0 = c + 13$ | $0 = c - 13$

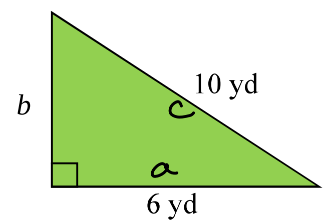
$c + 13 = 0$ | $c - 13 = 0$

$c = -13$ | $c = 13$

throw out -13 (it's a dimension)

$\boxed{\text{Missing side is } 13 \text{ in.}}$

7.



$a^2 + b^2 = c^2$

$6^2 + b^2 = 10^2$

$36 + b^2 = 100$

$b^2 = 64$

$b = \pm\sqrt{64}$

$b = \pm 8$

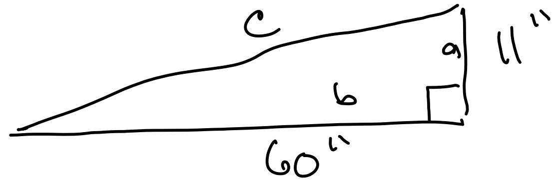
Throw out negative

$b = 8$ yds

$\boxed{\text{Missing side is } 8 \text{ yds}}$

8. The deck of a treadmill forms a right triangle with the floor. Find the length of the treadmill deck.

11 in.



$a^2 + b^2 = c^2$

$11^2 + 60^2 = c^2$

$121 + 3600 = c^2$

$3721 = c^2$

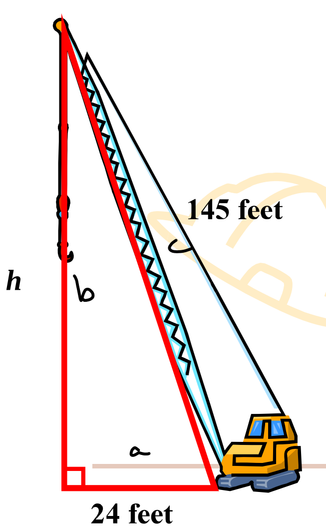
$c = \pm 61$

choose the positive.

$c = 61$ in

$\boxed{\text{The deck is 61 in long.}}$

9. Find the height of the crane.



24
×24
96
48
576

143
×143
429
572
143
20449

$a^2 + b^2 = c^2$

$h^2 + 24^2 = 145^2$

$h^2 + 576 = 21025$

$h^2 = 21025 - 576$

$h^2 = 20449$

$h = 143$

$\boxed{\text{The height is 143 ft.}}$