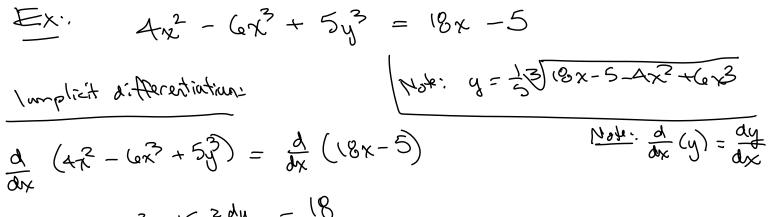
2.5: Implicit Differentiation

Example 1: Given the equation
$$x^{3} - 4y - 9x^{2} = 5$$
, find $\frac{dy}{dx}$ by
a) Solving explicitly for y.
b) Implicit differentiation.
a) Solut for y:
 $-4xy = 5 - x^{3} + 9x^{2}$
 $y_{\pm} = -\frac{5}{4} + \frac{1}{4}x^{3} - \frac{9}{4}x^{2}$
 $dy = -\frac{4}{4}\frac{dy}{dx} - 40x = 0$
 $3x^{2} - 4 + \frac{dy}{dx} - 40x = 0$
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 $3x^{2} - 4 + \frac{dy}{dx} - 40x = 0$
 $3x^{2} - 4 + \frac{dy}{dx} - 5x + 49x$
(implicit $dx^{2}f^{1}$.
 $7x + \frac{dy}{dx} = -\frac{3}{2}x^{2} + \frac{1}{2}x^{2}$
 $7x + \frac{1}{2}x + \frac{1}{2}x^{2} + \frac{1}{2}x^{2}$
 $7x + \frac{1}{2}x + \frac{1}{2}x^{2} + \frac{1}{2}x^{2}$
 $7x + \frac{1}{2}x + \frac{1}{2}x^{2} +$



$$8x - 18x^2 + 15y^2 dy = 18$$

Solve for
$$\frac{dy}{dx}$$
;
 $1 \le y^2 \frac{dy}{dx} = \frac{18 - 8x + 18x^2}{15x^2}$
 $\frac{dy}{dx} = \frac{18 - 8x + 18x^2}{15y^2}$

$$\frac{E \times \cdot \cdot}{\partial x} = 3x^{2} + y^{4} + \cos(y) = x^{17} - \sin x + y^{2}$$

$$\frac{d}{\partial x} (3x^{2} + y^{4} + \cos(y)) = \frac{d}{\partial x} (x^{17} - \sin(x) + y^{2})$$

$$(x + 4y^{3} \frac{dy}{dx} - (\sin(y)) \frac{dy}{dx} = 17x^{16} - \cos(x) + 2y \frac{dy}{dx}$$

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$$4y^{3} \frac{dy}{dx} - \frac{dy}{dx} \sin(y) - 2y \frac{dy}{dx} = (7x^{16} - \cos(x) - 6x)$$

$$Factor \quad \text{out} \quad \frac{dy}{dx}:$$

$$\frac{dy}{dx} (4y^{3} - \sin(y) - 2y) = (7x^{16} - \cos(x) - 6x)$$

$$\frac{dy}{dx} = \frac{(7x^{16} - \cos(x) - 6x)}{4y^{2} - \sin(y) - 2y}$$

Example 4: Find $\frac{dy}{dx}$ for the equation $\frac{1}{y^4} - \frac{5}{x^4} = 1$. $-\frac{4}{y^{5}}\cdot\frac{dy}{dx}=-\frac{20}{\chi^{5}}$ $y^{-4} - 5x^{-4} = 1$ $\frac{dy}{dx} = -\frac{20}{\chi^5} \left(-\frac{y^5}{4}\right)$ $\frac{d}{dx}\left(y^{2}-5x^{2}\right) = \frac{d}{dx}\left(1\right)$ $-4y^{-5}\frac{d}{dx}\left(y^{2}\right) + 20x^{-5} = 0$ $= \frac{20y^2}{4\sqrt{5}}$ $-\frac{1}{\sqrt{5}}\frac{dy}{d\chi}+\frac{20}{\chi^5}=0$ **Example 5:** Find $\frac{dy}{dx}$ for the equation $(x-y)^4 = y^2$. $\frac{d}{dx}\left(x-y\right)^{2}=\frac{d}{dx}\left(y^{2}\right)$ $4(x-y)^{2} \frac{d}{dx}(x-y) = 2y \frac{dy}{dx}$ 4 (x-y) (1- 24) = 24 and

$$4 (x - y)^{2} - 4(x - y)^{2} \frac{dy}{dx} = \lambda y \frac{dy}{dx}$$

Find $\frac{dy}{dy}$ for the equation $x + \cos(x^{2}y) = y$

Example 6: Find $\frac{dy}{dx}$ for the equation $x + \cos(x^2 y) = y$

$$\frac{d}{dx} \left(x + \cos(x^{2}y) \right) = \frac{d}{dx} \left(y \right)$$

$$\left| - \sin(x^{2}y) \frac{d}{dx} (x^{2}y) \right| = \frac{dy}{dx}$$

$$\left| - \sin(x^{2}y) \left(x^{2} \cdot \frac{d}{dx} (y) + y \frac{d}{dx} (x^{2}) \right) \right| = \frac{dy}{dx}$$

$$\left| - \sin(x^{2}y) \left(x^{2} \frac{dy}{dx} + y (2x) \right) \right| = \frac{dy}{dx}$$

$$\left| - x^{2} \sin(x^{2}y) \frac{dy}{dx} - 2xy \sin(x^{2}y) \right| = \frac{dy}{dx}$$

$$\left| - 2xy \sin(x^{2}y) \right| = \frac{dy}{dx} + x^{2} \sin(x^{2}y) \frac{dy}{dx} + x^{2} \sin(x^{2}y) \frac{dy}{dx}$$
See next
$$\left| - 2xy \sin(x^{2}y) \right| = \frac{dy}{dx} \left(1 + x^{2} \sin(x^{2}y) \right)$$

AN

$$\frac{dy}{dx} = \underbrace{\left[\frac{1 - 2xy}{4} \frac{4y}{2x} \frac{4y}{2x} \right]}_{(+ \sqrt{2} - 5! \wedge (\sqrt{2}y))} e_{0} \text{ isolut} \underbrace{dy}_{x} = \frac{-(+2xy - 5! \wedge (\sqrt{2}y))}{-(-\sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x} \frac{4y}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y)}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y)}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y)}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y)}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y))} \underbrace{\frac{1 - 2xy}{2x}}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y)}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y)}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2}y)}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2} - 5! \wedge (\sqrt{2}y)}_{(-1 - \sqrt{2} - 5! \wedge (\sqrt{2} - 5!$$

Example 7: Find $\frac{d^2 y}{dx^2}$ for the equation $x^3 - 2x^2 = y$. (*) find $\frac{d^2 y}{dx^2}$. $\frac{d}{dx} (x^3 - 2x^2) = \frac{d}{dx} (y)$ $3x^2 - 4x = \frac{dy}{dx}$ $\frac{d^2 y}{dx^2} = \frac{d}{dx} (3x^2 - 4x)$ $= \sqrt{(x^2 - 4x)}$

Example 8: Find
$$\frac{d^2 y}{dx^2}$$
 for the equation $xy^2 - y = 3$.
 $y = \frac{d}{dx} \left(\frac{d^2 y}{dx^2} + \frac{d}{dx} \left(\frac{d}{dx} - \frac{d}{dx} + \frac{d}{$

$$\frac{dy}{dx} = \frac{-y^2}{2xy - 1}$$

Quotient rule:

$$\frac{d}{dx} \begin{pmatrix} ay \\ ax \end{pmatrix} = \frac{(2xy-1)\frac{d}{ax}(-y^2) - (-y^2)\frac{d}{ax}(2xy-1)}{(2xy-1)^2} (auotient Rule)$$

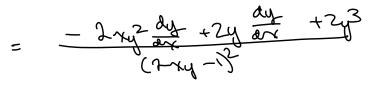
$$\frac{d}{dx} \begin{pmatrix} ay \\ ax \end{pmatrix} = \frac{(2xy-1)\frac{d}{ax}(-y^2) - (-y^2)\frac{d}{ax}(2xy-1)}{(2xy-1)^2} (auotient Rule)$$

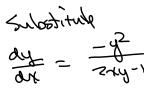
$$= \frac{(2xy-1)(-2y\frac{dy}{dx})+y^{2}(2x\frac{dy}{dx}+y\cdot 2)}{(2xy-1)^{2}}$$

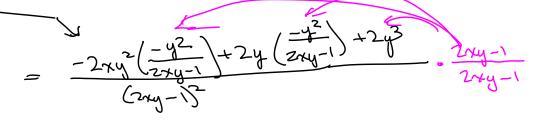
= $\frac{(2xy-1)^{2}}{(2xy-1)^{2}}$

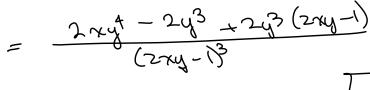
$$= - \frac{2}{\sqrt{y^2}} \frac{dy}{dx} + \frac{2y}{dx} + \frac{2y^3}{dx} + \frac{2y^3}{dx} + \frac{2y^3}{dx}$$

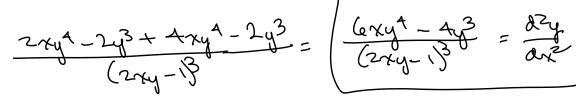
$$(2xy - 1)^2$$

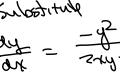












Example 9: Find the equation of the tangent line to the ellipse $4x^2 + 16y^2 = 64$ at the point $(2,\sqrt{3})$.

(2,v3).
Find
$$\frac{du}{dx}$$
 : $\frac{d}{dx} (4\chi + 1/4\chi) = \frac{d}{dx} (4\chi) + 1/4\chi = m(\chi - \chi)$
 $g_{\chi} + 32y \frac{dy}{d\chi} = 0$
 $g_{\chi} - 53 = -\frac{53}{6} (\chi - 1)$
 $g_{\chi} - \frac{53}{6} (\chi + \frac{53}{3} + \frac{53}{5})$
 $g_{\chi} - \frac{53}{6} (\chi + \frac{53}{3} + \frac{53}{5})$
Definition: Two curves are said to be *orthogonal* if at each point of intersection, their tangent
lines are perpendicular.

Example 10: Show that the hyperbola $x^2 - y^2 = 5$ and the ellipse $4x^2 + 9y^2 = 72$ are orthogonal.

Homework QS

$$2.4 \pm 29$$
 $g(x) = \frac{(x+5)^2}{(x^2+z)^2} = (x+5)^2 (x^2+z)^2$ use product rule
or $g(x) = \frac{(x+5)^2}{(x^2+z)^2}$ and use quotient rule
 $or g'(x) = 2(\frac{x+5}{x^2+z}) \frac{d}{dx}(\frac{x+5}{x^2+z})$ and use quotient rule
 2.4 ± 831 Telemine points in intruck (0,2vi) at which
graph of $f(x) = 2\cos x + \sin x$ has a vorizontal tanget.
Find $f'(x)$ and set t equal to 0.
Solve the equation (it's a trig egn)