5.2: The Natural Logarithmic Function: Integration

Using the derivative of the natural logarithmic function to obtain an antiderivative:

Example 1: Find the derivative of $g(x) = \ln|x|$. Note: domain: $x \neq 0$

$$g(x) = \ln|x| = \begin{cases} \ln x & \text{if } x > 0 \\ \ln(-x) & \text{if } x < 0 \end{cases}$$

$$g'(x) = \begin{cases} \frac{1}{x} & \text{if } x > 0 \\ \frac{1}{-x}(-1) = \frac{1}{x} & \text{if } x < 0 \end{cases}$$

So $g'(x) = \frac{1}{x}$ $\qquad \frac{d}{dx}(\ln|x|) = \frac{1}{x}$

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

Note that $f(x) = \ln x$ has the same derivative as $g(x) = \ln|x|$.

Therefore $\frac{d}{dx}\ln|x| = \frac{1}{x}$. This means that $f(x) = \ln|x|$ is an antiderivative of $F(x) = \frac{1}{x}$.

$$\int \frac{1}{x}\,dx = \ln|x| + c$$

Recall: The power rule for integrals $\int x^n dx = \frac{x^{n+1}}{n+1}$ had a restriction: $n \neq -1$. Now we can handle this case.

$$\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \ln|x| + c$$

Example 2: Determine $\int \frac{x^2}{x^3+4}\,dx$.

$u = x^3 + 4$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$\int \frac{1}{x^3+4} \cdot x^2\,dx$

$= \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C$

$= \frac{1}{3}\ln|x^3+4| + C$

Example 3: Determine $\int \frac{7}{2-5x}\,dx$.

Example 4: Determine $\int_2^5 \frac{1}{3x}\,dx$.

Example 5: Determine $\int \frac{x^7 - x + 3x^4}{x^5}\,dx$.

Example 6: Find $\int \frac{(\ln x)^4}{x}\,dx$.

Example 7: Find $\int \frac{\ln(3x)}{x}\,dx$.

Example 8: Find $\int \frac{x}{x^2-8}\,dx$.

Example 9: Find $\int \frac{4x^2-5x-12}{x^2-3}\,dx$.

<u>Example 10:</u> Find $\int \frac{4x^2 - 7x + 1}{2x - 3}\, dx$.

Integrating the remaining trigonometric functions:

<u>Example 11:</u> Determine $\int \tan x\, dx$.

<u>Example 12:</u> Determine $\int \cot x\, dx$.

Example 13: Determine $\int \sec x \, dx$.

Example 14: Determine $\int \csc x \, dx$.