Linear Equations in Two Variables Section 3.2

Definition of a Linear Equation in Two Variables

Definition Linear Equation in Two Variables: Let A, B, and C be real numbers such that A and B are not both zero. Then, an equation that can be written in the form: $Ax + By = C$ is called a **linear equation in two variables**.

Example: linear equation in two variables: $x + y = 5$

A solution to a linear equation in two variables is an ordered pair (x, y) that makes the equation a true equation.

For 1 and 2, determine whether the given ordered pair is a solution to the equation.

1. $y = \frac{1}{4}x + 5$; $(4,6)$ x y is a solution

$6 = \frac{1}{4}(\frac{4}{1}) + 5$ $6 = 6$ ✓

$6 = \frac{4}{4} + 5$

$6 = 1 + 5$

2. $x = 5$; $(2,5)$ x is not solution

$2 = 5$

Graphing Linear Equations in Two Variables by Plotting Points

Definition The Graph of an Equation in Two Variables: The graph of an equation in two variables is the graph of all ordered pair solutions to the equation.

The solution set for any linear equation in two variables forms a line in a rectangular coordinate plane.

To graph a linear equation in two variables: Find two solution points, and draw the line between them.

For exercises 3 – 5, complete each table and graph the corresponding ordered pairs. Draw the line defined by the points to represent all solutions to the equation.

3. $6x - 3y = 12$

x	y
0	-4
2	0
1	-2

1) $6(0) - 3y = 12$

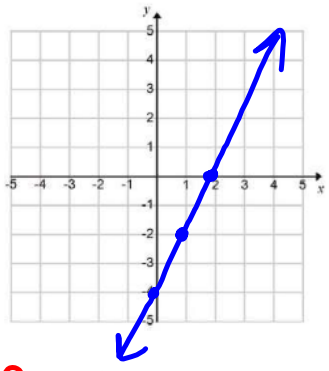
$\frac{-3y}{-3} = \frac{12}{-3}$

$y = -4$

2) $6x - 3(0) = 12$

$\frac{6x}{6} = \frac{12}{6}$

$x = 2$



3) $6(1) - 3y = 12$

$6 - 3y = 12$

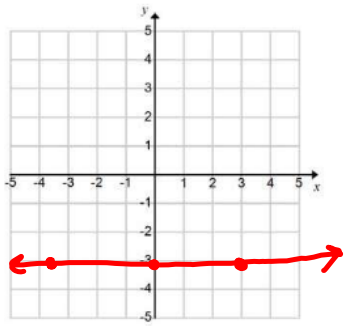
$-6 \quad -6$

$\frac{-3y}{-3} = \frac{6}{-3}$

$y = -2$

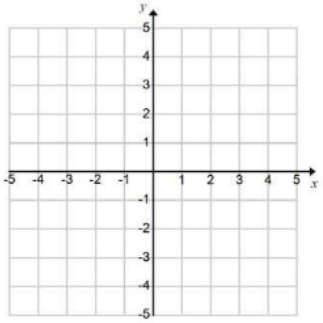
4. $y = -3$

x	y
0	-3
–4	-3
3	-3



5. $y = 0.8x + 1.2$

x	y
0	
1	
–2	



For exercises 6 and 7, graph the lines by making a table of at least three ordered pairs and plotting the points.

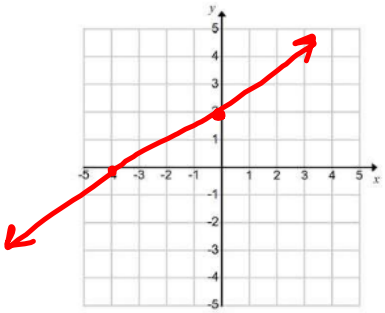
6. $-x + 2y = 4$

x	y
0	2
-4	0

$-0+2y=4$ $\quad -x+2(0)=4$

$\frac{2y}{2}=\frac{4}{2}$ $\quad \frac{-x}{-1}=\frac{4}{-1}$

$y=2$ $\quad x=-4$



7. $y = -\frac{1}{3}x + 2$

x	y
0	2
6	0

$y=-\frac{1}{3}(0)+2$

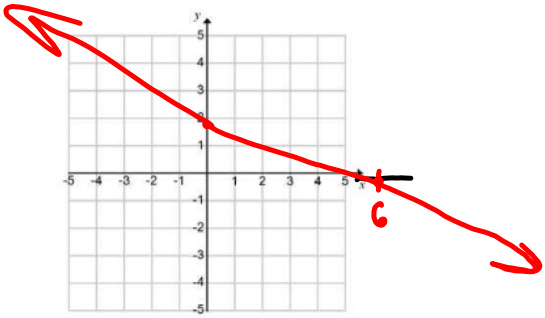
$y=2$

$0=-\frac{1}{3}x+2$

$-2 \quad -2$

$(-3)-2=-\frac{1}{3}\times\left(-\frac{3}{1}\right)$

$6=x$



x- and *y*-Intercepts

An *x*-intercept of a graph is a point $(a, 0)$ where the graph intersects the *x*-axis.

A *y*-intercept of a graph is a point $(0, b)$ where the graph intersects the *y*-axis.

To find the *x*- and *y*-intercepts

Step 1 Find the *x*-intercept(s) by substituting $y = 0$ into the equation and solving for *x*.

Step 2 Find the *y*-intercept by substituting $x = 0$ into the equation and solving for *y*.

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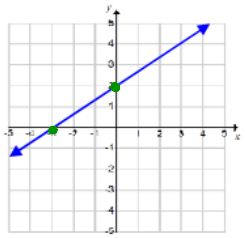
8. Estimate the coordinates of the *x*- and *y*-intercepts.



x-int: -3 (-3, 0)

y-int: 2 (0, 2)

For exercises 9 – 11 find the *x*- and *y*-intercepts (if they exist), and graph the line.

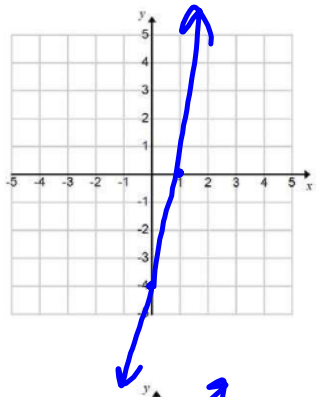
9. $4x - y = 4$

$4x - 0 = 4$

$\frac{4x}{4} = \frac{4}{4}$

$x = 1$

x	y	
0	-4	← y-int.
1	0	← x-int.



10. $y = \frac{3}{4}x + 3$

x	y
0	3
-4	0

$y = \frac{3}{4}(0) + 3$

$y = 3$

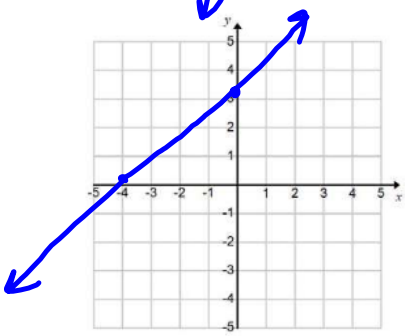
$0 = \frac{3}{4}x + 3$

$-3 \quad -3$

$\left(\frac{4}{3}\right)\left(-\frac{3}{1}\right) = \frac{3}{4}x\left(\frac{4}{3}\right)$

$-\frac{12}{3} = x$

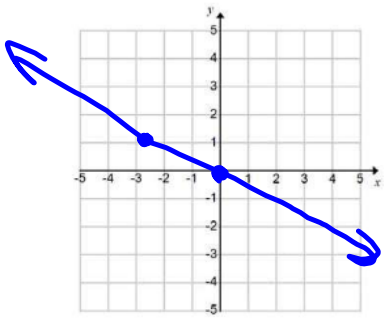
$-4 = x$



11. $x = -3y$

x	y
0	0
0	0
-3	1

$x = -3(1)$
$= -3$



Horizontal and Vertical Lines

A vertical line can be represented by an equation of the form, $x = k$, where k is a constant.

Examples of equations of vertical lines: $x = -3$, $x = \frac{1}{5}$, and $2x = -6$. no y variable

$x =$

A horizontal line can be represented by an equation of the form $y = k$, where k is a constant.

Examples of equations of vertical lines: $y = 0$, $y = 2.6$, and $y - 3 = 4$. no x variable

$y =$

For exercises 12 – 15, answer true or false. If the statement is false, rewrite it to be true.

y
x

12. The line $x = 5$ is a vertical line.

True

13. A line perpendicular to the y-axis is vertical.

False

14. A line parallel to the x-axis is horizontal.

True

15. Every line has both an x- and a y-intercept.

False

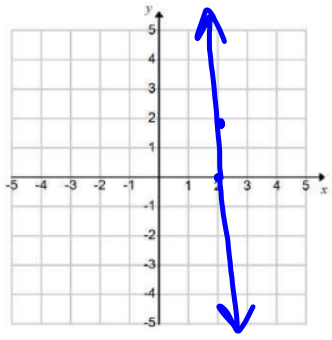
For exercises 16 –18, **a.** Identify the equation as representing a horizontal or vertical line.

b. Graph the line. **c.** Identify the x- and y-intercepts if they exist.

16. $x - 4 = -2$

$+4 \quad +4$

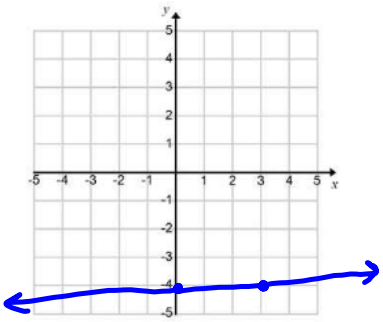
$x = 2$



17. $3y = -12$

$\frac{3}{3} \quad \frac{3}{3}$

$y = -4$



18. $\frac{1}{3}y = 0$

$\left(\frac{3}{1}\right)\frac{1}{3}y = 0\left(\frac{3}{1}\right)$

$y = 0$