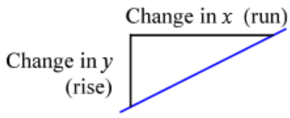
Slope of a Line and Rate of Change **Section 3.3**

Introduction to Slope

The slope of a line is the ratio of the vertical change (change in y) between two points and the horizontal change (change in x).

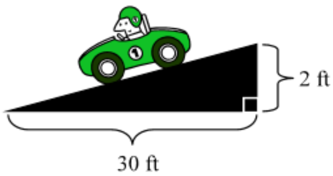
$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}$$



Geometrically, the slope of a line measures the "steepness" of the line.

1. Determine the slope of the road.

$$m = \frac{\text{rise}}{\text{run}} = \frac{2}{30} = \frac{1}{15}$$



Slope Formula

The slope of a line is often symbolized by the letter m.

The slope of a line passing through the distinct points (x_1, y_1) and (x_2, y_2) is

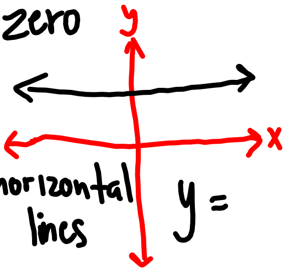
$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{provided } x_2 - x_1 \neq 0$$

When you apply the slope formula, you will see that the slope of a line is

- positive if the line increases, or rises, from left to right.
- negative if the line decreases, or falls, from left to right.
- zero if the line is horizontal.
- undefined if the line is vertical.

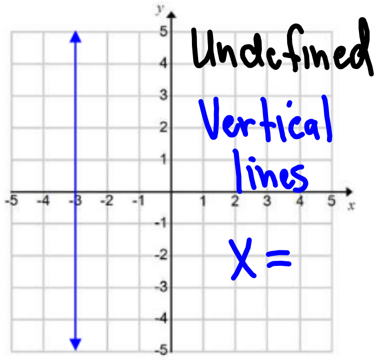
For exercises 2 and 3, determine if the slope is positive, negative, zero, or undefined.

zero



horizontal lines $y=$

2.

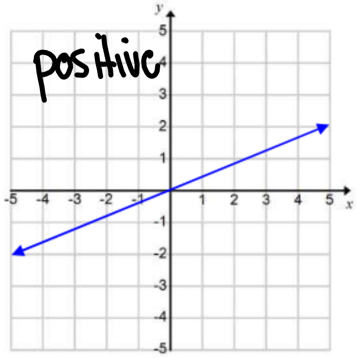


Undefined

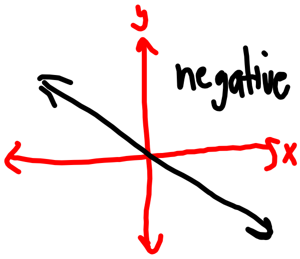
Vertical lines

$X=$

3.



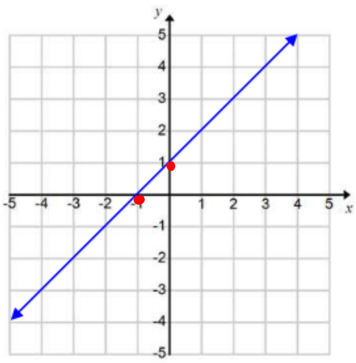
positive



negative

For exercises 4 and 5, determine the slope by using the slope formula and any two points on the line. Check your answer by drawing a right triangle and labeling the "rise" and "run".

4.



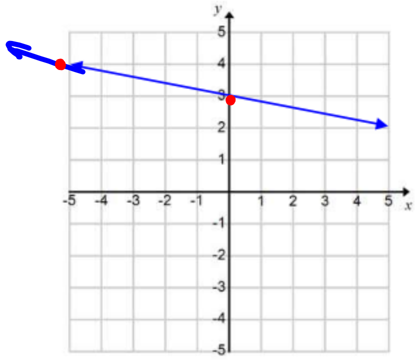
$(-1,0),(0,1)$

$x_1\ y_1 \quad x_2\ y_2$

$$m=\frac{1-0}{0+(+1)}=\frac{1}{1}=1$$

$$m=1$$

5.



$(-5,4),(0,3)$

$$m=\frac{3-4}{0+(+5)}=\frac{-1}{5}$$

$$m=-\frac{1}{5}$$

For exercises 6 – 9, find the slope of the line that passes through the two points.

6. $(2,-4)$ and $(5,7)$ (x_1 y_1 … x_2 y_2)

$$m = \frac{7-(-4)}{5-2}$$

7. $(-1,2)$ and $(-1,8)$

8. $\left(\frac{1}{2},\frac{2}{3}\right)$ and $\left(\frac{2}{5},-\frac{1}{4}\right)$

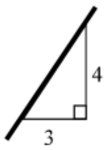
9. $(-3.1,4.5)$ and $(-2.5,6.3)$

Parallel and Perpendicular Lines

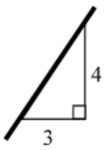
Parallel lines: Lines in the same plane that do not intersect.

Slopes of parallel lines: If m_1 and m_2 represent the slopes of two parallel (nonvertical) lines, then $m_1 = m_2$.

Parallel lines have the *same slope* and *different y-intercepts*.



$m_1 = \frac{4}{3}$



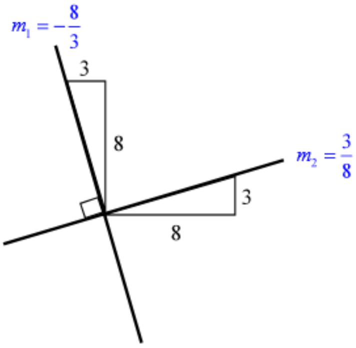
$m_2 = \frac{4}{3}$

Perpendicular lines: Lines that intersect at a right angle.

Slopes of perpendicular line: If $m_1 \neq 0$ and $m_2 \neq 0$ represent the slopes of two perpendicular lines, then

$$m_1 = -\frac{1}{m_2} \text{ or equivalently, } m_1 m_2 = -1.$$

If two lines are perpendicular then the slope of one line is the *opposite of the reciprocal* of the slope of the other line (provided neither line is vertical)



For exercises 10 – 12, the slope of the line is given.

a. Determine the slope of a line parallel to the given line.

b. Determine the slope of a line perpendicular to the given line.

10. $m = -3$

11. The slope is undefined

12. $m = \frac{1}{8}$

For exercises 13 – 15, let m_1 and m_2 represent the slopes of two lines. Determine if the lines are parallel, perpendicular or neither.

13. $m_1 = 4,\ m_2 = -\frac{1}{4}$

14. $m_1 = 2,\ m_2 = \frac{8}{4}$

15. $m_1 = \frac{2}{3},\ m_2 = \frac{3}{2}$

For exercises 16 and 17 find the slopes of the lines l_1 and l_2 defined by the two given points. Then determine whether l_1 and l_2 are parallel, perpendicular, or neither.

16. $l_1 : (3,6)$ and $(-2,16)$
    $l_2 : (-3,-10)$ and $(-2,-12)$

17. $l_1 : (1,7)$ and $(3,10)$
    $l_2 : (-4,10)$ and $(-7,12)$