4.2: The Substitution Method (for solving linear systems)

Note Title 12/1/2016

Example: Solve the system by substitution

$$\begin{cases} -4x+3y=19 \\ -6x-5y=0 \end{cases}$$

1) Solve $-6x-5y=0$ for y:

$+6x \quad +6x$

$$-5y = 6x$$

$$\frac{-5y}{-5} = \frac{6x}{-5}$$

$$y = -\frac{6}{5}x$$

2) Substitute $y = -\frac{6}{5}x$ into $-4x+3y=19$:

$$-4x + 3\left(-\frac{6}{5}x\right) = 19$$

3) Solve for x:

$$-4x - \frac{18}{5}x = 19$$

$$\frac{5}{5}\left(\frac{-4x}{1}\right) - \frac{18}{5}x = 19$$

$$-\frac{20}{5}x - \frac{18}{5}x = 19$$

$$-\frac{38}{5}x = 19$$

$$-\frac{5}{38}\left(-\frac{38}{5}x\right) = \frac{19}{1}\left(-\frac{5}{38}\right)$$

$$x = -\frac{5}{2}$$

4) Put $x = -\frac{5}{2}$ into $-6x-5y=0$ and solve.

see next page

$$-6x - 5y = 0$$

$$-6\left(-\frac{5}{2}\right) - 5y = 0$$

$$\frac{30}{2} - 5y = 0$$

$$15 - 5y = 0$$

$$15 = 5y$$

$$\frac{15}{5} = \frac{5y}{5}$$

$$3 = y$$

Solution: $\left(-\frac{5}{2}, 3\right)$

This is an independent system.

4.3: Solving Systems Using the Addition Method

Step-by-step for solving systems by the addition method (aka Elimination method)

1) multiply one or both equations by a strategic number, so that a variable will be eliminated when we add them.
2) Add the resulting equations.
3) Solve for the remaining variable.
4) Substitute that value into either of the original equations. solve.
   OR, repeat the addition process a 2nd time to eliminate the other variable.
5) Check your solution.

Ex: Solve the system

$$-x + 6y = 2$$
$$3x - 18y = 5$$

$$-x + 6y = 2 \xrightarrow{(3)} -3x + 18y = 6$$
$$3x - 18y = 5 \longrightarrow 3x - 18y = 5$$

Add $0x + 0y = 11$

$0 = 11$ False!

Inconsistent system
No solution
(lines are parallel)

Ex: Solve the system.

$$\begin{cases} 2x - 10y = 6 \\ -5x + 25y = -15 \end{cases}$$

$$2x - 10y = 6 \xrightarrow{(5)} 10x - 50y = 30$$
$$-5x + 25y = -15 \xrightarrow{(2)} -10x + 50y = -30$$
$$0x + 0y = 0$$
$$0 = 0 \quad \text{True!}$$

Dependent system
Infinitely many solutions
Solution Set: $\{(x,y) \mid 2x - 10y = 6\}$
(both equations represent the same lines)

How to recognize the dependent and inconsistent systems:

when solving by substitution or elimination, and both variables disappear:

1) If you get a false statement (such as $0 = 11$ or $5 = -7$) the system is inconsistent and has no solution (lines are parallel)

2) If you get a true statement (such as $0 = 0$ or $4 = 4$), then the system is dependent and has infinitely many solutions (lines are the same)