7.6: Rational Equations

Note Title 12/8/2016

<u>Definition:</u> An equation with one or more rational expressions is a <u>rational equation</u>.

Recall: a rational expression is a quotient (fraction) (ratio) of 2 polynomials.

<u>Example:</u> Solve.

$$\frac{4}{5} + \frac{2}{3}a = 2 - \frac{1}{5}a$$

multiply by the LCD to clear the fractions:

$$\frac{4}{\cancel{5}_1}\left(\frac{\cancel{15}^3}{1}\right) + \frac{2}{\cancel{3}_1}a\left(\frac{\cancel{15}^5}{1}\right) = 2(15) - \frac{1}{\cancel{5}_1}a\left(\frac{\cancel{15}^3}{1}\right)$$

$$\underset{+3a}{12 + 10a} = \underset{+3a}{30 - 3a}$$

$$\underset{-12}{12 + 13a} = \underset{-12}{30}$$

$$13a = 18$$

$$a = \frac{18}{13}$$

$$\boxed{\left\{\frac{18}{13}\right\}}$$

<u>Ex:</u> $\frac{2y-1}{4} + \frac{1}{6} = \frac{y}{2}$ LCD: 12

$$\left(\frac{2y-1}{\cancel{4}_1}\right)\left(\frac{\cancel{12}^3}{1}\right) + \frac{1}{\cancel{6}_1}\left(\frac{\cancel{12}^2}{1}\right) = \frac{y}{\cancel{2}_1}\left(\frac{\cancel{12}^6}{1}\right)$$

$$(2y-1)(3) + 2 = 6y$$

$$6y - 3 + 2 = 6y$$

$$\underset{-6y}{6y - 1} = \underset{-6y}{6y} \quad \text{False}$$

$$-1 = 0 \quad \text{False}$$

$\boxed{\text{No solution}}$

Example: Solve

LCD: $6d$

Restricted values: $d \neq 0$

$$\frac{3}{d} - \frac{5}{6} = \frac{1}{6}$$

$$\frac{3}{d}\left(\frac{6}{6}\right) - \frac{5}{6}\left(\frac{d}{d}\right) = \frac{1}{6}\left(\frac{d}{d}\right)$$

$$\frac{18}{6d} - \frac{5d}{6d} = \frac{d}{6d}$$

multiply by the LCD $6d$:

$$\frac{18}{\cancel{6d}}(\cancel{6d}) - \frac{5d}{\cancel{6d}}(\cancel{6d}) = \frac{d}{\cancel{6d}}(\cancel{6d})$$

$$18 - 5d = d$$

$$\quad +5d \quad +5d$$

$$18 = 6d$$

$$\frac{18}{6} = \frac{6d}{6}$$

$$3 = d$$

or

$$\frac{3}{d} - \frac{5}{6} = \frac{1}{6}$$

multiply by LCD:

$$\frac{3}{\cancel{d}}\frac{(6\cancel{d})}{1} - \frac{5}{\cancel{6}}\frac{(\cancel{6}d)}{1} = \frac{1}{\cancel{6}}\frac{(\cancel{6}d)}{1}$$

$$18 - 5d = 1d$$

$$\boxed{\text{Sol'n Set: } \{3\}}$$

Ex: Solve.

$$1 - \frac{3}{t} = \frac{28}{t^2}$$

LCD: t^2

Restricted values: $t \neq 0$

$$1(t^2) - \frac{3}{t}\frac{(t^2)}{1} = \frac{28}{\cancel{t^2}}\frac{(\cancel{t^2})}{1}$$

$$t^2 - \frac{3t}{1} = \frac{28}{1}$$

$$t^2 - 3t = 28$$

$$t^2 - 3t - 28 = 0$$

$$(t+4)(t-7) = 0$$

$t + 4 = 0$ | $t - 7 = 0$

$t = -4$ | $t = 7$

Check:

$(t+4)(t-7)$

$= t^2 - 7t + 4t - 28$

$= t^2 - 3t - 28$ ✓

$$\boxed{\text{Sol'n Set: } \{-4, 7\}}$$

Example: Solve.

LCD: $3(m+3)$

$$\frac{5m}{3m+9} + \frac{5}{m+3} = 1$$

Restricted values:

$m \neq -3$

$$\frac{5m}{3(m+3)} + \frac{5}{m+3} = 1$$

$$\frac{5m}{\cancel{3}\cancel{(m+3)}} \cdot \frac{\cancel{(3)}\cancel{(m+3)}}{1} + \frac{5}{\cancel{m+3}} \cdot \frac{(3)\cancel{(m+3)}}{1} = 1(3)(m+3)$$

$$5m + 15 = 3m + 9$$

$$-3m \qquad -3m$$

$$2m + 15 = 9$$

$$-15 \qquad -15$$

$$2m = -6$$

$$\frac{2m}{2} = \frac{-6}{2}$$

$$m = -3$$

Check this in the original: $\frac{5m}{3m+9} + \frac{5}{m+3} = 1$

$m = -3 \Rightarrow \frac{5(-3)}{3(-3)+9} + \frac{5}{-3+3} = 1$

$$\frac{-15}{-9+9} + \frac{5}{0} = 1$$

Cannot divide by 0!

$$\frac{-15}{0} + \frac{5}{0} = 1$$

Not true because $\frac{5}{0}$ and $\frac{15}{0}$ are not defined.

$m = -3$ does not check in the original eqn, so throw it out.

$\boxed{\text{No Solution}}$

Important: When solving an equation that has variables in the denominator, you <u>must check your solutions</u> in the original equation to see if they result in zero denominators.

<u>Extraneous solution</u> (really not a solution at all): value of the variable that is generated by a correct solution process, that does not make the original equation true.

Throw these values out – they're not in the solution set.

<u>Example</u>:

$$\frac{9}{x+3} = \frac{x}{x+3} + 5 \qquad \text{LCD: } x+3$$

<u>Ex</u>:

$$\frac{x^2+5x}{x-2} = \frac{14}{x-2}$$

Done in class

LCD: $(x+3)(x-3)$

<u>Ex</u>:

$$\frac{x}{x+3} = \frac{8}{x^2-9} + 2$$

$$\frac{x}{x+3} = \frac{8}{(x+3)(x-3)} + 2$$

$$\frac{x}{x+3}\cdot\frac{(x+3)(x-3)}{1} = \frac{8}{(x+3)(x-3)}\cdot\frac{(x+3)(x-3)}{1} + 2(x+3)(x-3)$$