# 4.5: Conditional Probability

**Conditional probability:**

Consider the probability that a house will be flooded during a given year. Would you expect this probability to be different if you only considered houses that were located in a 50-year flood plain?

**Example 1:** Draw a single card from a standard 52-card deck.

$S$ = set of 52 cards
$J$ = set of jacks

a. What is the probability that you draw a jack?

$P(J) = \frac{4}{52} = \boxed{\frac{1}{13}}$

b. New information….Given that you drew a face card (K,Q,J), what is the probability that it is a jack?

$F$ = set of face cards
$n(F) = 3(4) = 12$

So the prob. of getting a jack given it's a face card is $\frac{4}{12} = \boxed{\frac{1}{3}}$. We call this probability $P(J|F)$.

"given"
4/52

Notation: $P(A|B)$ denotes the probability of $A$ given that $B$ occurs.

"the probability of the intersection divided by the probability of the given"

| Conditional probability definition: |
| --- |
| The probability of $A$ given that $B$ occurs is |
| $$P(A\|B) = \frac{P(A \cap B)}{P(B)}. \qquad (P(B) \neq 0)$$ |

c. Use the conditional probability definition to determine the probability that a jack is drawn, given that the card is a face card.

$P(J \cap F) =$
$P(F) = 12/52$

$$P(J|F) = \frac{P(J \cap F)}{P(F)} = \frac{4/52}{12/52}$$

$$= \frac{4}{\cancel{52}} \cdot \frac{\cancel{52}}{12} = \frac{4}{12} = \boxed{\frac{1}{3}}$$

**Example 2:** Draw a single card from a standard 52-card deck. What is the probability of drawing the ace of diamonds given that the card is red?

S = set of red cards

$n(S) = 2(13) = 26$

Only 1 ace of diamonds, so

$P(A\diamond|R) = \boxed{\frac{1}{26}}$

**Example 3:** In a test conducted by the U.S. Army, it was found that of 1000 new recruits, 680 men and 320 women, 57 of the men and 3 of the women were red-green color-blind. Given that a recruit selected at random from this group is red-green color-blind, what is the probability that the recruit is a male?

B: red-green color-blind recruits

m: men

$$P(m|B) = \frac{P(m \cap B)}{P(B)} = \frac{57/1000}{60/1000} = \frac{57}{60} = \boxed{\frac{19}{20}}$$

OR

$$\frac{57}{60} = \boxed{\frac{19}{20}}$$

**Example 4:** This table shows the number of adult men and women with diabetes in 2012.
http://www.cdc.gov/diabetes/pubs/statsreport14/national-diabetes-report-web.pdf
Tables like these are called *contingency tables* or *two-way tables*.

| | Diabetics (in millions) | Non-diabetics (in millions) | Total |
|---|---|---|---|
| Men | 15.5 | 98.5 | 114.0 |
| Women | 13.4 | 106.2 | 119.6 |
| Total | 28.9 | 204.7 | 233.6 |

a. What is the probability that a randomly selected adult is diabetic, given that the person is male?

$$\frac{15.5}{114} \approx \boxed{0.136}$$

b. What is the probability that a randomly selected adult is diabetic?

$$\frac{28.9}{233.6} \approx \boxed{0.124}$$

c. What is the probability that a randomly selected adult is a diabetic female?

$$\frac{13.4}{233.6} \approx \boxed{0.057}$$

**Example 5:**

**4.142 Living Arrangements.** As reported by the U.S. Census Bureau in *America's Families and Living Arrangements*, the living arrangements by age of U.S. citizens 15 years of age and older are as shown in the following joint probability distribution.

| | | Living arrangement | | | |
|---|---|---|---|---|---|
| | | Alone $L_1$ | With spouse $L_2$ | With others $L_3$ | $P(A_i)$ |
| Age (yr) | 15–24 $A_1$ | 0.006 | 0.012 | 0.157 | 0.175 |
| | 25–44 $A_2$ | 0.030 | 0.184 | 0.123 | 0.337 |
| | 45–64 $A_3$ | 0.047 | 0.216 | 0.067 | 0.330 |
| | Over 64 $A_4$ | 0.046 | 0.088 | 0.024 | 0.158 |
| | $P(L_j)$ | 0.129 | 0.500 | 0.371 | 1.000 |

A U.S. citizen of age 15 or older is selected at random. Determine the probability that the person selected:

a) lives alone.
b) is under 45 and lives alone.
c) is over 64 and does not live with a spouse.
d) lives alone, given that the person is 45 or older.
e) What percentage of people over 64 live alone?
f) What percentage of people living with a spouse are under 25?