4.6: The Multiplication Rule; Independence

Independence of events:

Two events are said to be *independent* of the outcome of one does not affect the outcome of the other. If they are not independent, then they are said to be *dependent*.

> Independent Events:
> Events A and B are independent events if and only if:
>
> - $P(A|B) = P(A)$ or, equivalently,
> - $P(B|A) = P(B)$ or, equivalently,
> - $P(A \cap B) = P(A)P(B)$.

$P(A|B) = \dfrac{P(A\cap B)}{P(B)}$

mult by $P(B)$:

$P(A|B)P(B) = P(A\cap B)$

Independent, so $P(A|B)=P(A) \Rightarrow P(A)P(B) = P(A\cap B)$

Example 1: This table shows the number of adult men and women with diabetes in 2012. Does diabetes seem to be independent of gender?
http://www.cdc.gov/diabetes/pubs/statsreport14/national-diabetes-report-web.pdf

D: has diabetes

	Diabetics (in millions)	Non-diabetics (in millions)	Total
Men	15.5	98.5	114.0
Women	13.4	106.2	119.6
Total	28.9	204.7	233.6

Let's see whether $P(D|m) = P(D)$.

$$P(D|m) = \frac{15.5}{114} \approx 0.136$$

$$P(D) = \frac{28.9}{233.6} \approx 0.124$$

Hmmm.... are these numbers close enough to the same to be considered independent?

You would need to do a statistical test to answer that.

Example 2: A survey conducted found that of 2000 women, 680 were heavy smokers and 50 had emphysema. Of those who had emphysema, 42 were also heavy smokers. Using this data, determine whether the events "being a heavy smoker" and "having emphysema" were independent events.

H: heavy smoker

E: has emphysema

Are $P(E|H) = P(E)$?

$$P(E|H) = \frac{P(E \cap H)}{P(H)} = \frac{42}{680} \approx 0.062$$

$$P(E) = \frac{50}{2000} \approx 0.025.$$

Emphysema and heavy smoking are not independent, because $0.025 \neq 0.062$.

Example 3: Suppose that a basketball player has a 78% free throw percentage, and that ~~his~~ the outcome of one free throw attempt does not affect his next attempt. If he attempts two free throws, what is the probability he makes both of them?

Here, we use $P(A \cap B) = P(A)P(B)$, if A and B are independent.

S: success on 1 free throw attempt

$P(S) = 0.78$

Attempt #1 and attempt #2 are independent, so

$$P(S_1 \cap S_2) = P(S_1)P(S_2)$$
$$= 0.78\,(0.78) = \boxed{0.6084}$$

Independence of more than two events:

> If $E_1, E_2, \ldots, E_n$ are independent, then
>
> $$P(E_1 \cap E_2 \cap \ldots \cap E_n) = P(E_1)P(E_2)\ldots P(E_n).$$

Example 4: Suppose that a basketball player has a 78% free throw percentage, and that his the outcome of one free throw attempt does not affect his next attempt. Suppose he attempts five free throws.

a) What is the probability he makes all five?
b) What is the probability that he makes the first three, misses one, and then makes the last?
c) What is the probability that he misses at least one of the five free throws?

a) $P(S_1 \cap S_2 \cap S_3 \cap S_4 \cap S_5) = P(S_1)P(S_2) \ldots P(S_5)$

$= 0.78(0.78)(0.78)(0.78)(0.78)$

$= (0.78)^5 = \boxed{0.2887}$

b) $P(\text{miss}) = P(S^c) = 1 - 0.78 = 0.22$

$0.78(0.78)(0.78)(0.22)(0.78) = (0.78)^4(0.22) = \boxed{0.0814}$

c) From part (a), there is a 0.2887 prob. he makes all 5. So the prob ~~of~~ missing at least 1 is $1 - 0.2887 = \boxed{0.7113}$

Example 5: A certain loudspeaker system has four components: a woofer, a midrange, a tweeter, and an electrical crossover. It has been determined that on the average 1% of the woofers, 0.8% of the midranges, 0.5% of the tweeters, and 1.5% of the crossovers are defective. Determine the probability that a randomly chosen loudspeaker is not defective. Assume that the defects in the different types of components are unrelated.

$P(\text{nondefective woofer}) = 1 - 0.01 = 0.99$

$P(\text{nondefective midrange}) = 1 - 0.008 = 0.992$

$P(\text{nondefective tweeter}) = 1 - 0.005 = 0.995$

$P(\text{nondefective crossover}) = 1 - 0.015 = 0.985$

$P(\text{nondefective system}) = 0.99(0.992)(0.995)(0.985) = \boxed{0.9225}$

Example 6: An auto repair company tracks the customers who return to the shop for a repetition of a problem that was previously repaired. If the repeat problem is due to a technician error or a defective part, the customer will not be charged again. Company records show that about 1.5% of catalytic converters are defective, and that the technician makes an error in about 2% of the catalytic converter installations. What is the probability that a catalytic converter customer ends up repeating the repair free of charge?

I: Installation error

D: defective part

We want to find $P(I \cup D)$

Basic counting/addition formula for probability (section 4.3)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(I \cup D) = P(I) + P(D) - P(I \cap D).$$

We know

$P(I) = 0.02$

$P(D) = 0.015$

Assuming I and D are independent, we know that

$$P(I \cap D) = P(I)P(D).$$

$$P(I \cup D) = P(I) + P(D) - P(I)P(D) = 0.02 + 0.015 - 0.02(0.015) = \boxed{0.0347}$$

The product rule (multiplication rule) for intersections of events:

Recall: The definition for conditional probability:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

multiply both sides by $P(A)$: $P(B|A)P(A) = P(A \cap B)$

Product rule:

If A and B are events with nonzero probabilities, then

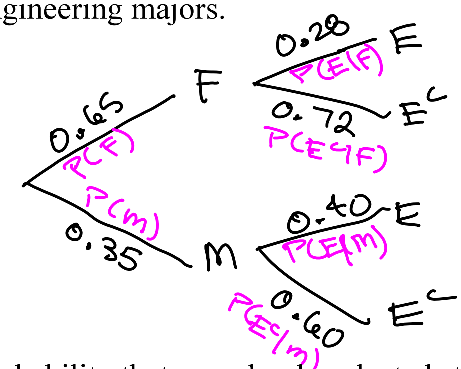
$$P(A \cap B) = P(B \mid A)P(A).$$

Probability trees:

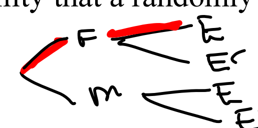
The product rule for conditional probability allows us to set up probability trees.

<u>Example 7:</u> In a certain class, 65% of the students are female. 40% of the males and 28% of the females are engineering majors.

E: Engineering major

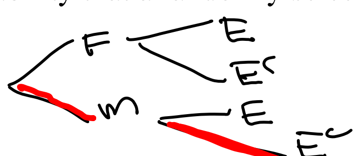


a) What is the probability that a randomly selected student is female and an engineering major?



$P(E\cap F) = 0.65\,(0.28) = \boxed{0.182}$

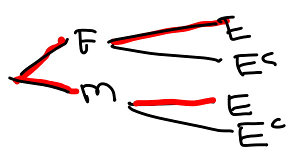
b) What is the probability that a randomly selected student is male and a non-engineering major?



$P(m \cap E^c) = 0.35\,(0.60)$

$= \boxed{0.21}$

c) What is the probability that a randomly selected student is an engineering major?



$P(E) = 0.65(0.28) + 0.35\,(0.40)$

$= \boxed{0.322}$

<u>Example 8:</u> A certain type of camera is manufactured in three locations. Plants A, B, and C supply 45%, 30%, and 25%, respectively, of the cameras. The quality-control department of the company has determined that 1.5% of the cameras produced by plant A, 2% of the cameras produced by plant B and 2.75% of the cameras produced by plant C are defective. What is the probability that a randomly selected camera is defective?

Example 9: A medical study found that about 8% of screening mammograms resulted in a "false positive" (the test indicated possible cancer when cancer was not present).
http://www.breastcancer.org/research-news/false-positives-may-be-linked-to-higher-risk
(159,448+ 22,892)/2,207,942
About 20% of screening mammograms result in a "false negative" (the test appeared normal, even though cancer was present).
https://www.cancer.gov/types/breast/mammograms-fact-sheet

a) Use a probability tree to illustrate the probabilities of false negatives and false positives.
b) Based on these probabilities, what is the probability of a screening mammogram indicating possible cancer?
c) Based on these probabilities, what is the probability that a screening mammogram gives a correct result?