6.3: Working with Normally Distributed Variables

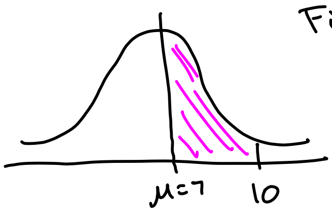
Recall: The *z*-score of a data point is its distance from the mean, measured in standard deviations.

> Standardizing the values of a normal distribution:
>
> In a normal distribution with mean μ and standard deviation σ, where *x* is a data value, the *z*-score is
>
> $$z = \frac{x-\mu}{\sigma}.$$
>
> The area under a normal curve between $x = a$ and $x = b$ is the same as the area under the standard normal curve between the *z*-score for *a* and the *z*-score for *b*.

Example 1: Consider a normal curve with mean 7 and standard deviation 2.

$\mu = 7,\ \sigma = 2$

a) What is the area under the curve between 7 and 10?



Find z-score for $x=10$:

$$z = \frac{x-\mu}{\sigma} = \frac{10-7}{2} = \frac{3}{2} = 1.50$$

Look up $z = 1.50$ in table $\Rightarrow$ $\boxed{\text{Area} = 0.4332}$

Part (b)

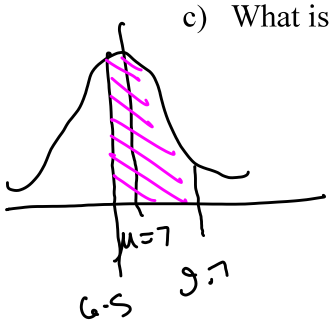
$P(7 < X < 10)$

$= P(0 < Z < 1.50)$

$= \boxed{0.4332}$

b) What is the probability that the variable is between 7 and 10?

c) What is the probability that the variable is between 6.5 and 9.7?



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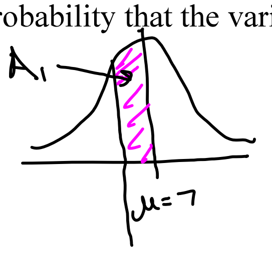

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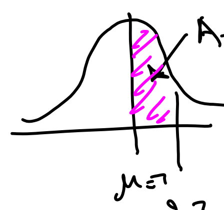


z-score for 6.5 is

$$z = \frac{x-\mu}{\sigma} = \frac{6.5-7}{2} = -0.25$$

From table, $z = 0.25$

$\Rightarrow A_1 = 0.0987$

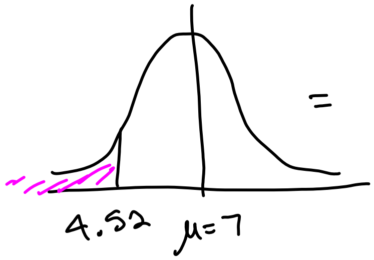
z-score for 9.7 is

$$z = \frac{x-\mu}{\sigma} = \frac{9.7-7}{2} = 1.35$$

From table, $A_2 = 0.4115$

$P(6.5 < X < 9.7) = P(-0.25 < Z < 1.35) = A_1 + A_2 = 0.0987 + 0.4115 = \boxed{0.5102}$

d) What is the probability that the variable is less than 4.52?



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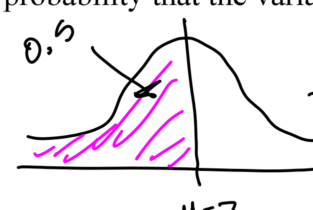

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z-score for 4.52 is

$$z = \frac{x-\mu}{\sigma} = \frac{4.52-7}{2} = -1.24$$

From Table, $A_1 = 0.3925$

$P(X < 4.52) = P(Z < -1.24) = 0.5 - A_1$

$= 0.5 - 0.3925 = \boxed{0.1075}$

<u>Properties of Normal Probability Distributions</u>:

1. $P(a \le x \le b) =$ area under the curve from a to b.
2. $P(-\infty \le x \le \infty) = 1 =$ total area under the curve.
3. $P(x = c) = 0$.

Note: $P(a \le x \le b) = P(a \le x < b) = P(a < x \le b) = P(a < x < b)$

Example 2: Dusty Dog Food Company ships dog food to its distributors in bags whose weights are normally distributed with a mean weight of 50 pounds and standard deviation 0.5 pound. If a bag of dog food is selected at random from a shipment, what is the probability that it weighs

a) More than 51 pounds?
b) Less than 49 pounds?
c) Between 49 and 51 pounds?
d) What is the percentage of dog food bags that weigh more than 51 pounds?

$\mu = 50, \quad \sigma = 0.5$

ⓒ Find prob bag is between 49 and 51 lbs

Find z-score for 51: $z = \frac{x-\mu}{\sigma} = \frac{51-50}{0.5} = 2.00$

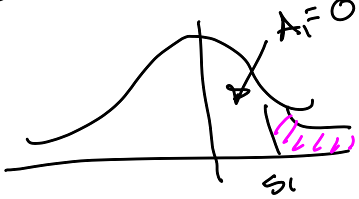
From symmetry, $A_1 = A_2$

From Table,
$A_1 =$ Area $= 0.4772$

$P(49 < x < 51) = 2A_1 = 2(0.4772)$
$= \boxed{0.9544}$

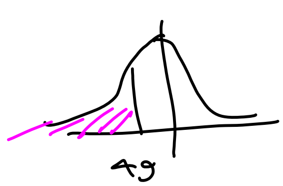
ⓐ $P(x > 51) = 0.5 - 0.4772 = \boxed{0.0228}$

$A_1 = 0.4772$



ⓑ $P(x < 49) = \boxed{0.0228}$

(from symmetry, same as $P(x > 51)$)



ⓓ Percentage weighing > 51 lbs:

$\boxed{2.28\%}$

<u>Example 3:</u> The medical records of infants delivered at a certain hospital show that the infants' birth weights in pounds are normally distributed with a mean of 7.4 and a standard deviation of 1.2.

a) What percentage of infants at this hospital weighed more than 9.2 pounds at birth?
b) What percentage of infants at this hospital weighed less than 8 pounds at birth?
c) What percentage of infants at this hospital weighed between 8 and 10 pounds at birth?

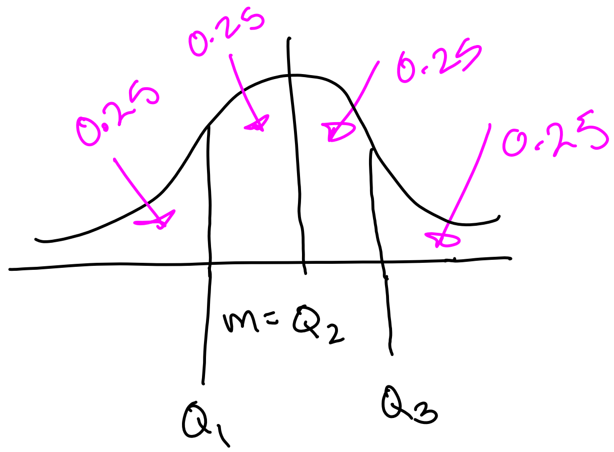
Important: The *z*-score is the number of standard deviations between the data point and the mean.

Example 4: A variable is normally distributed with mean 83 and standard deviation 24.

$\mu = 83, \quad \sigma = 24$

a) Find and interpret the quartiles.
b) Find and interpret the 98th percentile.
c) Find and interpret the first and second deciles.
d) Find the value that 72% of all possible values of the variable exceed.
e) Find two values of the variable that divide the area into a middle area of 0.90 and two outside areas of 0.05 each.

(a)



$Q_2 = \mu = \text{median} = 83$
$Q_3 = 99.2$
$Q_1 = 66.8$

So $Q_3 \Rightarrow z_{0.25}$

Look up area = 0.25 in table

It's between A = 0.2486 and A = 0.2517

So $z_{0.25} \approx 0.675$

(halfway between $z = 0.67$, $z = 0.68$)

Find the x-value:

$$z = \frac{x - \mu}{\sigma}$$

multiply by σ: $z\sigma = x - \mu$

Add μ: $z\sigma + \mu = x$

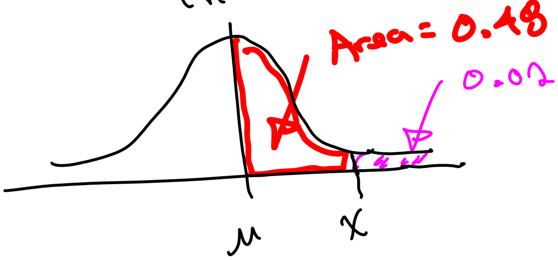
$$x = \mu + z\sigma = 83 + 0.675(24) = 99.2$$

Find Q_1:

$$x = \mu - z\sigma = 83 - 0.675(24) = 66.8$$

(b) 98th percentile:

This means 2% are above, and 98% are below



Look up Area = 0.48 in table

$z \approx 2.055$

$$x = \mu + z\sigma = 83 + 2.055(24) = \boxed{132.32}$$

(c) 1st decile: 10% are below, so 90% are above

2nd decile: 20% are below, so 80% are above

etc

<u>Example 5:</u> The GPA of the senior class of a certain high school is normally distributed with a mean of 2.7 and a standard deviation of 0.4 point. If a senior in the top 10% of his or her class is eligible for admission to any state university, what is the minimum GPA that a senior should have to ensure eligibility to a state university?