## 9.5: Hypothesis Tests for One Population Mean When σ is Unknown

In practice, when we are using a sample to make inferences about the population mean, it is rare for us to know the population standard deviation.

Instead, we must use the sample standard deviation, *s*, as a point estimate of the population standard deviation, $\sigma$.

When using *s* as an estimate for $\sigma$, we cannot use a *z*-test, because $\dfrac{\overline{x} - \mu}{\dfrac{s}{\sqrt{n}}}$ is not normally distributed.

**The *t*-test for one population mean:**

When using *s* as an estimate for $\sigma$, we use the Student *t*-distribution.

In order to use this procedure, we need to know (or be able to reasonably assume) that the variable of interest follows a normal distribution, or we must have a large sample size ($n \geq 30$).
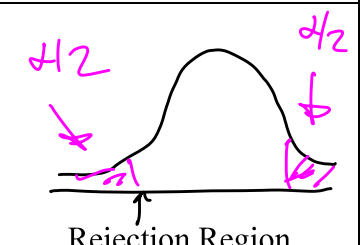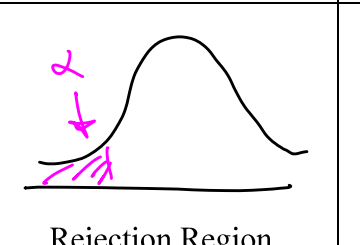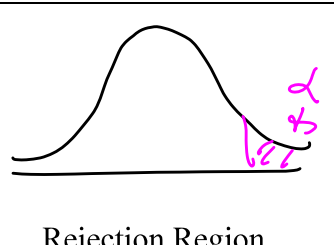
In addition, the sample should be randomly obtained, observations within the sample must be independent of one another. This means that if we have a sample size that is more than 5% of the population, we should multiply the standard error by a finite population correction factor, $\sqrt{\dfrac{N-n}{n-1}}$. (In this class, I do not anticipate that we will encounter this situation.)

**Hypothesis Testing for a Population Mean:**

Step 1: Determine the significance level $\alpha$ .

Step 2: Determine the null and alternative hypotheses.

| Two-Tailed Test (most common) | Left-Tailed Test (rare) | Right-Tailed Test (rare) |
|---|---|---|
| $H_0 : \mu = \mu_0$  $H_1 : \mu \neq \mu_0$ | $H_0 : \mu = \mu_0$  $H_1 : \mu < \mu_0$ | $H_0 : \mu = \mu_0$  $H_1 : \mu > \mu_0$ |
| Rejection Region | Rejection Region | Rejection Region |

Note: One tailed tests assume that the scenario not listed ( $\mu > \mu_0$ for a left-tailed test or $\mu < \mu_0$ for a right-tailed test)  is not possible or is of zero interest.

Step 3: Use your $\alpha$ level and hypotheses, sketch the rejection region.

Step 4: Compute the test statistic $t = \dfrac{\overline{x} - \mu}{\dfrac{s}{\sqrt{n}}}$ .

Step 5: Use a table (Table IV, on page A-13) to determine the critical value for $t$ associated with your rejection region.

Step 6: Determine whether the value of $t$ calculated from your sample (in Step 3) is in the rejection region.
- If $t$ is in the rejection region, reject the null hypothesis.
- If $t$ is not in the rejection region, do not reject the null hypothesis.

Step 7: State your conclusion.

**Example 1:** The normal human body temperature is widely accepted to be $98.6°\,F$ and can be assumed to follow a normal distribution. A medical researcher wants to know whether a certain geographical community of Native Alaskans ~~has a mean~~ *differs from* body temperature ~~of~~ $98.6°\,F$. A sample of 20 members of the Native Alaskan geographical community resulted in a mean body temperature of $98.3°\,F$ with a standard deviation of $0.7°\,F$. Perform an appropriate hypothesis test at the 95% confidence level.

null:      $H_0: \mu = 98.6°F$

alternative:    $H_a: \mu \neq 98.6°F$

$$\alpha = 1 - 0.95 = 0.05$$

**Sample info**

$n = 20$
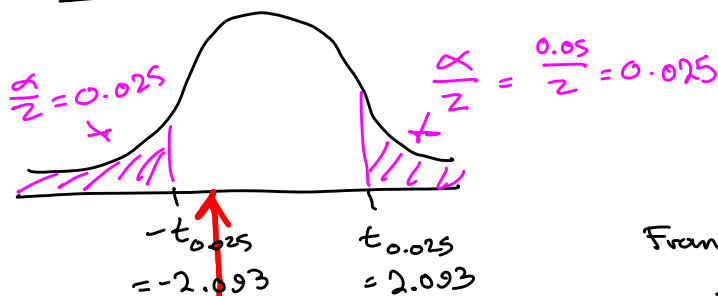
$df = n - 1 = 20 - 1 = 19$

$\bar{x} = 98.3°F$

$s = 0.7°F$

Find critical value of $t$:



$$\frac{\alpha}{2} = 0.025$$

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

$-t_{0.025} = -2.093$     $t_{0.025} = 2.093$

$t = -1.92$

Calculate $t$ for our sample:

standard error: $\sigma_{\bar{x}} \approx \dfrac{s}{\sqrt{n}} = \dfrac{0.7}{\sqrt{20}}$

$$= 0.1565$$

From table, for $df = 19$,

$$t_{0.025} = 2.093$$

$$t = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{98.3 - 98.6}{0.1565}$$

$$= -1.92$$

This is **not** in the rejection region, so we **Do not reject** $H_0$

This sample does not provide evidence that the temp. differs from $98.6°F$.

**Example 2:** The average amount of lead in the blood of young children is 2 micrograms per deciliter (mcg/dL). A city has recently changed its water supply, and there have been widespread reports of increased lead levels in the water. A concerned doctor wants to dig into the city's medical records to find out whether the children in the city have blood lead levels above the average level of 2 mcg/dL. In a sample of 35 children, she found a mean lead level of 2.60 mcg/dL with a standard deviation of 1.9 mcg/dL. Perform an appropriate hypothesis test at the 95% confidence level.

$H_0: \mu = 2\ mcg/dL$

$H_a: \mu > 2\ mcg/dL$

$\alpha = 1 - 0.95 = 0.05$

Find critical value of $t$



$\alpha = 0.05$

$t_{0.05} = 1.691$

$t = 1.87$

**Sample info:**

$n = 35$

$df = n - 1 = 35 - 1 = 34$

$\bar{x} = 2.60\ mcg/dL$

$s = 1.9\ mcg/dL$

From table, for $df = 34$,

$$t_{0.05} = 1.691$$

$t = 1.87$ is in rejection region, so we **Reject** $H_0$.

Calculate $t$ for our sample

std. error: $\sigma_{\bar{x}} \approx \dfrac{s}{\sqrt{n}} = \dfrac{1.9}{\sqrt{35}}$

$$= 0.32116$$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{2.60 - 2}{0.32116}$$

$$= 1.87$$

This sample provides evidence the children's lead levels are higher than normal.

$\star$ Benchmark
$\mu$

**Example 3:** Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. A customer (or a FDA analyst) buys 50 bags of chips, weighs them on a high-accuracy scale, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance.
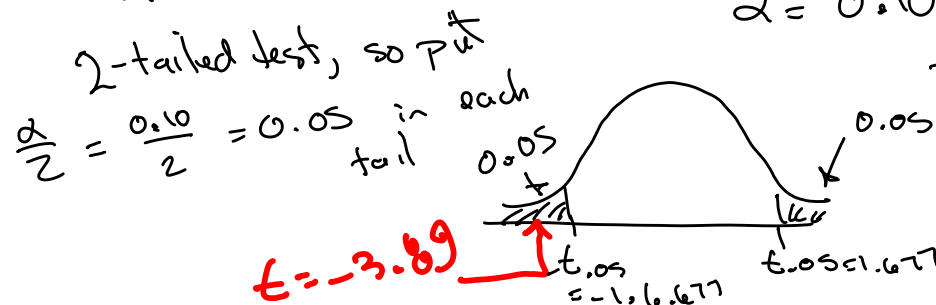
$\bar{x} = 7.89$

$s = 0.20$

$n = 50$

$df = n - 1 = 49$

2-tailed test, so put

$\frac{\alpha}{2} = \frac{0.10}{2} = 0.05$ in each tail

$H_0: \mu = 8$

$H_a: \mu \neq 8$

$\alpha = 0.10$

std error: $\frac{s}{\sqrt{n}} = \frac{0.2}{\sqrt{50}}$

$= 0.002828$

$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{7.89 - 8}{0.002828} = -3.89$

From table, for $df = 49$,

$t_{0.05} = 1.677$

$t = -3.89$ in rejection region, so we reject $H_0$

(sample provides evidence that the mean is not 8.00z)

$t = -3.89 \to \downarrow t_{.05} = -1.6.677$     $0.05$     $0.05$     $t_{.05} = 1.677$
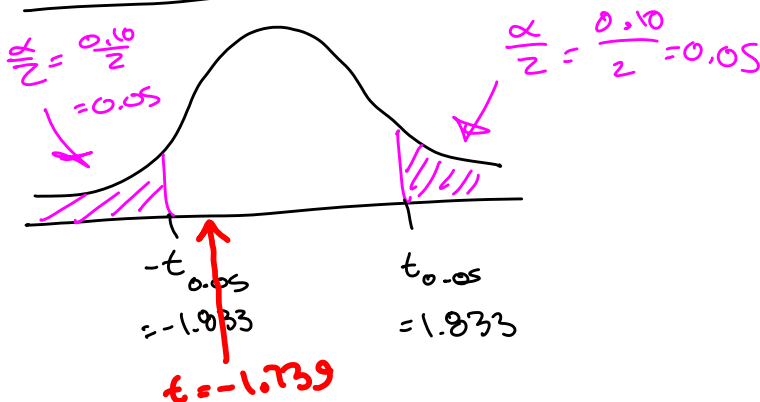
**Example 4:** Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. Again, a customer (or a FDA analyst) wonders whether the package size is accurate. This time, the analysist only buys 10 bags of chips, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance.

$\alpha = 0.10$

$H_0: \mu = 8 oz$

$H_a: \mu \neq 8 oz$

Find critical value of t

$\frac{\alpha}{2} = \frac{0.10}{2} = 0.05$      $\frac{\alpha}{2} = \frac{0.10}{2} = 0.05$

$-t_{0.05} = -1.833$      $t_{0.05} = 1.833$

$t = -1.739$

From table, for $df = 9$,

$t_{0.05} = 1.833$

Do not reject $H_0$;

This sample does not provide evidence the mean weight differs from 8oz

Sample info:

$n = 10$,

$df = 10 - 1 = 9$

$\bar{x} = 7.89$

$s = 0.2$

To do this test, we would need to assume the weights are normally distributed (since sample size < 30)

Calculate t for our sample:

std error: $\sigma_{\bar{x}} \approx \frac{s}{\sqrt{n}} = \frac{0.2}{\sqrt{10}}$

$\approx 0.06325$

$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{7.89 - 8}{0.06325} = -1.739$