Supplement: Basic Set Theory

Definition: A *set* is a well-defined collection of objects. Each object in a set is called an *element* of that set.

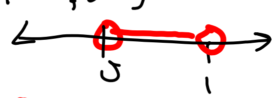
Examples of sets:
set of odd positive integers
set of students enrolled in this class
U.S. citizens over 6 ft tall

Not sets:
All tall people
All cute dogs

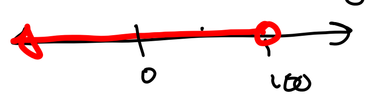
Sets can be finite or infinite.

Examples of finite sets: set of positive integers less than 100: $\{1,2,3,4,\ldots,98,99\}$
students enrolled at LSC-NH
All people living in the U.S.

Examples of infinite sets:
set of integers less than 100: $\{\ldots,-2,-1,0,1,2,3,\ldots,99\}$
Interval of real numbers between 0 and 1 $(0,1)$



Set of real numbers below 100: $(-\infty,100)$



Notation:
- We usually use capital letters for sets.
  We usually use lower-case letters for elements of a set.
  $a \in A$
- $a \in A$ means *a* is an element of the set *A*.
  $a \notin A$ means *a* is not an element of the set *A*.
  $a \notin A$
- The *empty set* is the set with no elements. It is denoted $\varnothing$. This is sometimes called the *null set*.
- $S = \{x \mid P(x)\}$ means "*S* is the set of all *x* such that $P(x)$ is true". (called rule notation or set roster notation). "such that"

  Example: $S = \{x \mid x \text{ is an even positive integer}\}$ means $S = \{2,4,6,8,\ldots\}$
  $S = \{(x,y) \mid x>0, y>0\}$ = 1st quadrant of xy-plane
- $n(A)$ means the number of elements in set *A*.

Definition: We say two sets are *equal* if they have exactly the same elements.

Subsets:

Definition: If each element of a set A is also an element of set B, we say that A is a *subset* of B. This is denoted $A \subseteq B$ or $A \subset B$. If A is not a subset of B, we write $A \not\subset B$.

$A \subseteq B$ or $A \subset B$

Definition: We say A is a *proper subset* of B if $A \subseteq B$ but $A \neq B$. (In other words, every element of A is also an element of B, but B contains at least one element that is not in A.)

Note on notation: Some books use the symbol $\subset$ to indicate a proper subset. Some books use $\subset$ to indicate any subset, proper or not.

or universe

Definition: The set of all elements under consideration is called the *universal set*, usually denoted U.

Example: If you're dealing with sets of real numbers, then U is the set of all real numbers. So "Wednesday" would not be an element of U, but 5.7 would be in U.

Example 1: Consider these sets.

$$A = \{1,2,3,4,5,6\}$$
$$B = \{1,2,3,4,5,6,7,8\}$$
$$C = \{1,3,5,2,4,6\}$$

$A = C$

$A \subseteq B$

Also $C \subseteq B$

Note:

- $\varnothing$ is a subset of every set. (i.e. $\varnothing \subseteq A$ for every set A.)
- Every set is a subset of itself. (i.e. $A \subseteq A$ for every set A.)

The empty set

Example 2: List all subsets of $\{1,2,3\}$.

$\{1,3\}$, $\{1,2\}$, $\{2,3\}$, $\{2\}$, $\{1\}$, $\{3\}$, $\{1,2,3\}$, $\varnothing$

Note: If a set has n elements, how many subsets does it have?

Turns out it has 2^n subsets (always)

So, $\{1,2,3\}$ has $n=3$. It has $2^3 = 8$ subsets.

Set operations:

- Union $\cup$: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

  Key word: OR

  our book has (A or B)

- Intersection $\cap$: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

  Key word: AND

  our book has A & B

- Complement A' or A^C or $A^{\sim}$: $A' = \{x \in U \mid x \notin A\}$.

  Key word: NOT

  (our book says (NOT A)

Note: $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$.
$(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$.

→ mutually exclusive

Definition: We say that A and B are *disjoint sets* if $A \cap B = \varnothing$.
(if their intersection is empty)

Example 3: $U = \{1,2,3,4,5,6,7,8\}$
$H = \{1,3,5,7\}$
$K = \{1,2,3\}$
$J = \{2,4,6,8\}$
$L = \{1,2\}$

$H \cap K = \{1,3\}$
(must be in both H and K)

$H \cup K = \{1,3,5,7,2\}$
(must be in H or K or possibly both)

$J \cup K = \{2,4,6,8,1,3\}$

K-complement: $K' = K^c = \{4,5,6,7,8\}$

$J^c = \{1,3,5,7\} = H$

Venn Diagrams: These help us visualize set relationships and operations.

Example 4: Draw Venn diagrams for $A \cup B$, $A \cap B$, A^C, B^C, $(A \cap B)^C$, and $(A \cup B)^C$.