1.6: Properties of Real Numbers & Simplifying Expressions

Review: Properties of Real Numbers

Commutative Property of Addition: $a+b=b+a$

(Commutative means we can change the order)

Ex: $2+3=3+2$ (both equal 5)

Commutative Property of Multiplication: $ab=ba$

Note: $ab = a \cdot b = a(b) = (a)(b)$
all these notations mean multiplication.

Do not use $\times$ for multiplication!

Ex: $5 \cdot 7 = 7 \cdot 5$
(both equal 35)

Note: Division and subtraction are not commutative (cannot switch the order)

Ex: $\frac{8}{2} \neq \frac{2}{8}$

$\frac{8}{2} = 4$, $\frac{2}{8} = \frac{1}{4}$

Ex: $12-5 \neq 5-12$

$12-5=7$

$5-12=-7$

Note: $-12+5=-7$

$5+{}^{-}12=-7$

Associative Property of Addition:

(Associative means you can regroup)

$a+(b+c)=(a+b)+c$

Ex: $2+(3+4)=(2+3)+4$
(Both equal 9

Associative Property of Multiplication: $a(bc) = (ab)c$

Ex: $2(3\cdot4) = (2\cdot3)\cdot4$
(both equal 24)

Distributive Property of Real Numbers: $a(b+c) = ab+ac$
$(a+b)\cdot c = ac+bc$

(multiplication distributes over addition)

Ex: $2(3+4)$
$= 6+8 = \boxed{14}$

Ex: $2+(3+4)$
$= 2+7$
$= 9$

Ex: $+2(3+4)$
$= 6+8 = 14$

Ex: $-2(3+4)$
$= -6 - 8$
$= \boxed{-14}$

or
$-2(3+4)$
$= -2(7)$
$= \boxed{-14}$

Review of Signed numbers

$-5+3 = -2$
$-5-3 = -8$
$5-3 = 2$
$3-5 = -2$
$-3+5 = 2$

same

same

$-5(3) = -15$
$-5(-3) = 15$
$5(-3) = -15$

Algebraic Expression: Numbers and variables combined with arithmetic operations

(Note: An expression does not have an equals sign.

Examples of expressions: $2x+5$

$3x^2y$

Algebraic expressions being added are called terms.
Algebraic expressions being multiplied are called factors.
The coefficient is the numerical factor in a term.

Ex: $3x^2 - 7x + 5$ has 3 terms.

The coefficient of the x^2 term is 3.
The coefficient of the x term is -7.
The constant (constant coefficient) is 5.

When simplifying, we combine like terms by adding the coefficients.

(Like terms have the same variables & exponents).

Ex: $2x^2, 5x^2, -8x^2, \frac{2}{3}x^2$ are all like terms

Ex: $4x^2, 7x$ are not like terms.

Ex: Simplify.

Ex: $\underline{2x^2} - \underline{8x} + \underline{\underline{5}} - \underline{3x^2} - \underline{\frac{1}{2}x} + \underline{\underline{17}}$

$-1x^2 - 8x - \frac{1}{2}x + 22$

$-1x^2 - \frac{8x}{1}\left(\frac{2}{2}\right) - \frac{1}{2}x + 22$

$-1x^2 - \frac{16x}{2} - \frac{1}{2}x + 22$

$-x^2 - \frac{16}{2}x - \frac{1}{2}x + 22 = \boxed{-x^2 - \frac{17}{2}x + 22}$

Note: We usually stick to improper fractions (ex: $\frac{17}{2}$) instead of switching to mixed numbers (Ex: $8\frac{1}{2}$) unless there's a good reason to switch (reasons = dimensions, graphing)

Note: $\frac{7x}{2} = \frac{7}{2}x$

Why? $\frac{7}{2}x = \frac{7}{2} \cdot \frac{x}{1} = \frac{7x}{2}$

Caution: Never write these: $\frac{7}{2}\quad x$

$7/2x$

(we cannot tell if the writer intended the x to be in the numerator or the denominator)

Ex: $\frac{2}{3}\left(4x - \frac{1}{4}\right) - \frac{3}{8}\left(\frac{1}{2}x - \frac{1}{3}\right)$

$\frac{2}{3}\left(\frac{4x}{1}\right) + \frac{2}{3}\left(-\frac{1}{4}\right) - \frac{3}{8}\left(\frac{1}{2}x\right) - \frac{3}{8}\left(-\frac{1}{3}\right)$

$\frac{8x}{3} - \frac{\cancel{2}^{1}}{\cancel{12}_{6}} - \frac{3}{16}x + \frac{\cancel{3}^{1}}{\cancel{24}_{8}}$

$\frac{8}{3}x - \frac{1}{6} - \frac{3}{16}x + \frac{1}{8}$

$\frac{8}{3}x - \frac{3}{16}x - \frac{1}{6} + \frac{1}{8}$

$\frac{8}{3}x\left(\frac{16}{16}\right) - \frac{3}{16}x\left(\frac{3}{3}\right)$

$-\frac{1}{6}\left(\frac{4}{4}\right) + \frac{1}{8}\left(\frac{3}{3}\right)$

$\frac{128}{48}x - \frac{9}{48}x - \frac{4}{24} + \frac{3}{24}$

$\boxed{\frac{119}{48}x - \frac{1}{24}}$