

2.8: Linear Inequalities (cont'd)

Note Title

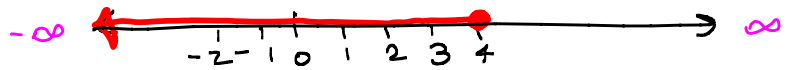
2/8/2017

Review Problems:

1) $x \leq 4$

Set builder: $\{x | x \leq 4\}$

Interval notation: $(-\infty, 4]$



2) $x > -1$

Set builder: $\{x | x > -1\}$

Interval notation:

$(-1, \infty)$



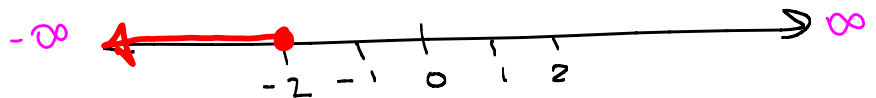
3) $-2 \geq x$

Rewrite: $x \leq -2$

Set-builder: $\{x | x \leq -2\}$

Interval notation:

$(-\infty, -2]$

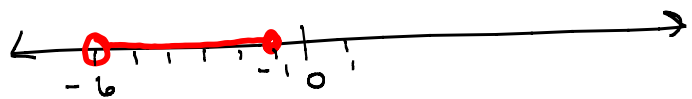


4) $-6 < x \leq -1$

Set-builder: $\{x | -6 < x \leq -1\}$

Interval notation:

$(-6, -1]$



Solving linear inequalities

Definition: A linear inequality is one that can be rearranged in the form $ax + b < c$, $ax + b > c$, $ax + b \leq c$, $ax + b \geq c$.

(a, b, c are numbers and x is the variable)

Note: all x 's to 1st power

Examples of linear inequalities

$$2x - 3 < 9, \quad x \leq 5(x - 17)$$

To solve an inequality means to find the solution set (the set of values that make it true).

Addition Property of Inequality

If $a < b$, then $a + c < b + c$.

This means we can add (or subtract) the same quantity on both sides.

Example: solve. $x - 7 < 12$

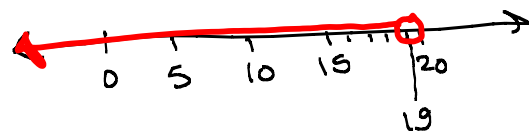
+7 +7

$$x < 19$$

Set-builder: $\{x \mid x < 19\}$

Interval:

$(-\infty, 19)$



Multiplicative Property of Inequality

If $a < b$ and $c > 0$, then $ac < bc$.

If $a < b$ and $c < 0$, then $ac > bc$.

This means: (or divide)

* we can multiply both sides by the same positive number.

* we can multiply (or divide) both sides by the same negative number if we reverse the inequality sign.

Why must we reverse the sign when multiplying/dividing by a negative?

$$1 < 2 \quad \text{True}$$

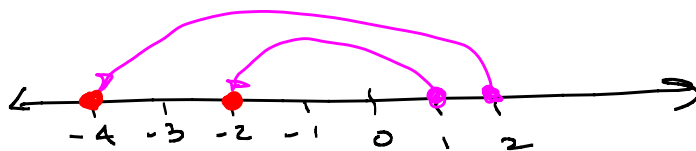
multiply both sides by 2:

$$1(2) < 2(2) \\ 2 < 4 \quad \text{still true!}$$

Start with $1 < 2$ again. ~~True~~

multiply both sides by -2 :

$$1(-2) < 2(-2) \\ -2 < -4 \quad \text{False!}$$



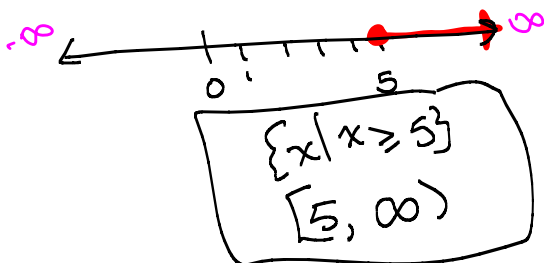
To make sure we have an equivalent (still true) inequality, we must reverse the inequality sign.

Example: Solve.

$$4x \geq 20$$

$$\frac{4x}{4} \geq \frac{20}{4}$$

$$x \geq 5$$

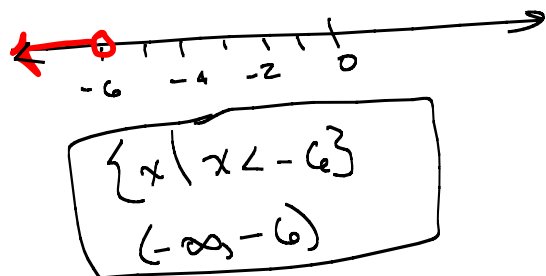


Ex: Solve.

$$-3x > 18$$

$$\frac{-3x}{-3} < \frac{18}{-3} \quad \text{Reverse the inequality}$$

$$x < -6$$



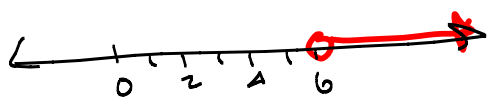
Ex: Solve.

$$-3x < -18$$

Reverse
inequality
sign

$$\frac{-3x}{-3} > \frac{-18}{-3}$$

$$x > 6$$



Interval:

$$(6, \infty)$$

Set-builder:

$$\{x | x > 6\}$$

Ex: Solve

$$3x < -18$$

$$\frac{3x}{3} < \frac{-18}{3}$$

$$x < -6$$



Interval:

$$(-\infty, -6)$$

Set-builder:

$$\{x | x < -6\}$$

Ex: $2 \leq 6x + 32$

-32

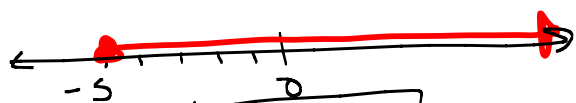
-32

$$-30 \leq 6x$$

$$\frac{-30}{6} \leq \frac{6x}{6}$$

$$-5 \leq x$$

Rewrite: $x \geq -5$



Interval:

$$[-5, \infty)$$

Set-builder:

$$\{x | x \geq -5\}$$

Ex: $-\frac{2}{3}x > \frac{5}{4}$

$$-\frac{3}{2} \left(-\frac{2}{3}x \right) < \frac{5}{4} \left(-\frac{3}{2} \right)$$

Reverse
the
inequality
sign

$$x < -\frac{15}{8}$$

$$\{x | x < -1\frac{7}{8}\}$$



$$(-\infty, -1\frac{7}{8})$$

Example: Solve.

$$\frac{2}{3}x - \frac{2}{5} < -7 + \frac{1}{2}x$$

$$\frac{2}{3}x - \frac{1}{2}x < -7 + \frac{2}{5}$$

$$\left(\frac{2}{2}\right) \frac{2}{3}x - \frac{1}{2}x \left(\frac{3}{3}\right) < -\frac{7}{1} \left(\frac{5}{5}\right) + \frac{2}{5}$$

$$\frac{4}{6}x - \frac{3}{6}x < -\frac{35}{5} + \frac{2}{5}$$

$$\frac{1}{6}x < -\frac{33}{5}$$

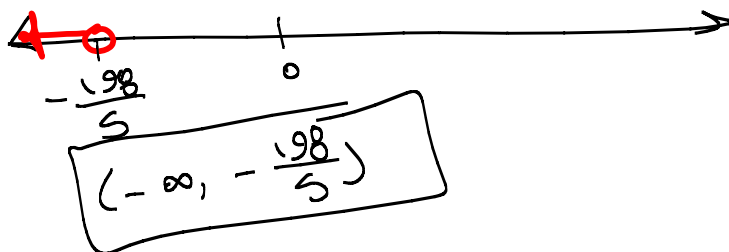
$$\left(\frac{6}{1}\right) \frac{1}{6}x < -\frac{33}{5} \left(\frac{6}{1}\right)$$

$$x < -\frac{198}{5}$$

$$x < -39\frac{3}{5}$$

$$\begin{array}{r} 33 \\ 6 \\ \hline 198 \end{array}$$

$$\begin{array}{r} 39 \\ 5 \overline{) 198} \\ \underline{15} \\ 48 \\ \underline{45} \\ 3 \end{array}$$



Combined inequalities:

Example: $-7 \leq 2x + 5 < 3$

This means: $-7 \leq 2x + 5$ and $2x + 5 < 3$
 $\quad \quad \quad -5 \quad \quad -5 \quad \quad \quad -5 \quad -5$

$$-12 \leq 2x$$

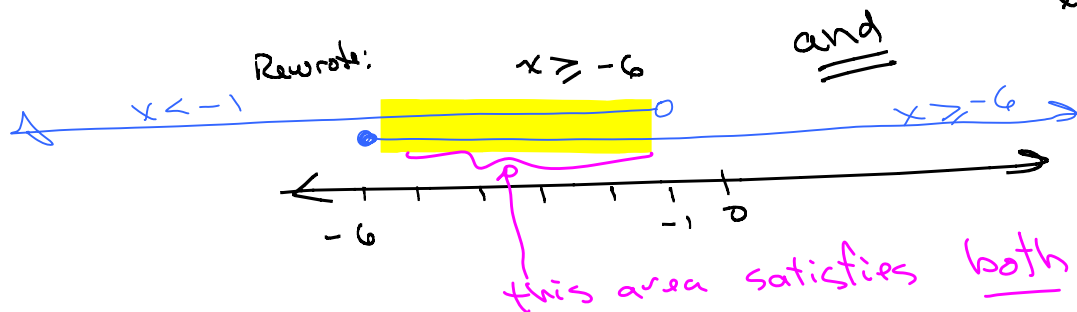
$$2x < -2$$

$$-\frac{12}{2} \leq \frac{2x}{2}$$

$$\frac{2x}{2} < \frac{-2}{2}$$

$$-6 \leq x$$

$$x < -1$$



Recombine $x \geq -6$ and $x < -1$

$$-6 \leq x \text{ and } x < -1$$

$$-6 \leq x < -1$$

Set builder:

$$\{x \mid -6 \leq x < -1\}$$

Interval:

$$[-6, -1)$$

OR
(Keep them together)

$$-7 \leq 2x + 5 < 3$$

$\quad \quad \quad -5 \quad \quad -5 \quad \quad -5$ [subtract 5 from all 3 sides]

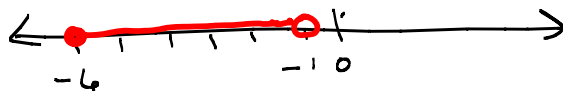
$$-12 \leq 2x < -2$$

$$-\frac{12}{2} \leq \frac{2x}{2} < \frac{-2}{2}$$

[Divide all 3 sides by 2]

$$-6 \leq x < -1$$

Set builder: $\{x \mid -6 \leq x < -1\}$
Interval: $[-6, -1)$



Example: Solve

$$-4 < 5(3-x) \leq 1$$

$$\begin{array}{ccccccc} -4 & < & 15 & - & 5x & \leq & 1 \\ & & -15 & & -15 & & -15 \end{array}$$

$$-19 < -5x \leq -14$$

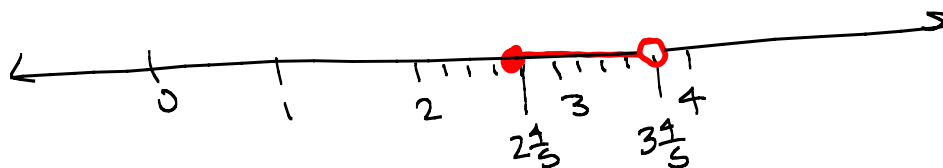
$$\frac{-19}{-5} > \frac{-5x}{-5} \geq \frac{-14}{-5}$$

[Reverse both inequality signs]

$$\frac{19}{5} > x \geq \frac{14}{5}$$

Rewrite: $\frac{14}{5} \leq x < \frac{19}{5}$

$$2\frac{4}{5} \leq x < 3\frac{4}{5}$$



$$\left[2\frac{4}{5}, 3\frac{4}{5}\right)$$

$$\{x \mid 2\frac{4}{5} \leq x < 3\frac{4}{5}\}$$