5.1: Multiplying and Dividing Expressions with Common Bases (Properties of Exponents)

Note Title 2/15/2017

Recall: An exponent indicates repeated multiplication.

Example: $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 9 = \boxed{81}$

Here, 3 is called the base and 4 is the exponent.

(exponent is often called a power)

Know these powers:

$2^2 = 4$	$3^2 = 9$	$5^2 = 25$
$2^3 = 8$	$3^3 = 27$	$5^3 = 125$
$2^4 = 16$	$3^4 = 81$	$5^4 = 625$
$2^5 = 32$	$3^5 = 243$	$6^2 = 36$
$2^6 = 64$	$3^6 = 729$	$6^3 = 216$
$2^7 = 128$	$4^2 = 16$	$7^2 = 49$
$2^8 = 256$	$4^3 = 64$	$7^3 = 343$
$2^9 = 512$	$4^4 = 256$	$8^2 = 64$
$2^{10} = 1024$		$8^3 = 512$
$10^2 = 100$	$9^2 = 81$	
$10^3 = 1000$	$9^3 = 729$	
$10^4 = 10\,000$		
$\vdots$		

$10^9 = 1\,000\,000\,000$

$= 1{,}000{,}000{,}000$

Powers of Negative Numbers

Example: $(-5)^2 = (-5)(-5) = \boxed{25}$

$-5^2 = -(5)(5) = \boxed{-25}$

Example: $(-2)^6 = \underbrace{(-2)(-2)}_{+}\underbrace{(-2)(-2)}_{+}\underbrace{(-2)(-2)}_{+} = \boxed{64}$

$(-2)^7 = \underbrace{(-2)(-2)}_{+}\underbrace{(-2)(-2)}_{+}\underbrace{(-2)(-2)}_{+}\underbrace{(-2)}_{-} = \boxed{-128}$

Important:

* If you raise a negative number to an <u>even</u> exponent, the result is <u>positive</u>.

* If you raise a negative number to an <u>odd</u> exponent, the result is <u>negative</u>.

Example: $(-3)^4 = \boxed{81}$

$(-5)^3 = \boxed{-125}$

$(-1)^{17} = \boxed{-1}$

$-1^{18} = \boxed{-1}$

$(-1)^{18} = \boxed{1}$

$-(-2)^4 = \boxed{-16}$

$-(-4)^3 = -(-64) = \boxed{64}$

Examples: $\left(-\frac{2}{3}\right)^4 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) = +\frac{16}{81}$

$= \boxed{\frac{16}{81}}$

Properties of Exponents

1) $x^a x^b = x^{a+b}$

Ex: $x^3 x^4 = (xxx)(xxxx) = x^7$

2) $(x^a)^b = x^{ab}$

Ex: $(x^2)^3 = (xx)^3 = (xx)(xx)(xx) = x^6$

3) $\frac{x^a}{x^b} = x^{a-b}$

Ex: $\frac{x^7}{x^3} = \frac{xxxxxxx}{xxx} = \frac{x^4}{1} = x^4$

4) $(xy)^a = x^a y^a$

Ex: $(xy)^3 = (xy)(xy)(xy) = xxxyyy = x^3y^3$

5) $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

Ex: $\left(\frac{x}{y}\right)^3 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{x^3}{y^3}$

Note: Our book covers Property #1 and #3 in 5.1, and the others in 5.2.

Example: $\frac{3^{11}}{3^9} = 3^{11-9} = \frac{3^2}{1} = \boxed{9}$

Ex: $\frac{2^5}{3^3} = \boxed{\frac{32}{27}}$

Ex: $(6a^4)(-3a^2) = 6(-3)a^4a^2$

$= \boxed{-18a^6}$

Ex: $(3r^5)(-4r^2)(r) = 3(-4)r^5r^2r^1$

$= -12r^{5+2+1} = \boxed{-12r^8}$

Ex: $\dfrac{a^3a^{14}}{a^4a^6} = \dfrac{a^{17}}{a^{10}} = \dfrac{a^7}{1} = \boxed{a^7}$

Ex: $\dfrac{27m^5n^{12}}{6m^4n^7} = \dfrac{\cancel{27}^{9}\, mn^5}{\cancel{6}_{2}} = \boxed{\dfrac{9mn^5}{2}} = \boxed{\dfrac{9}{2}mn^5}$

5.2: More Properties of Exponents

(Here, our book adds the properties

$(x^a)^b = x^{ab}$ ← Power Rule (power to a power you multiply)

$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

$(xy)^a = x^a y^a$)

Ex: $(z^6)^8 = \boxed{z^{48}}$

Ex: $(d \cdot d^4)^6$

$= (d^5)^6$

$= \boxed{d^{30}}$

Ex: $(5x^4)^2$

$= 5^2 (x^4)^2$

$= \boxed{25x^8}$

Ex: $(-2ab^3)^4$

$= (-2)^4 a^4 b^{12}$

$= \boxed{16a^4b^{12}}$

Ex: $(xy^2z^3)^5$

$x^5 (y^2)^5 (z^3)^5$

$= \boxed{x^5 y^{10} z^{15}}$

Ex: $\dfrac{(-3ab^3)^3(a^5b^2)^5}{(3a^2b^4c)^2}$

$= \dfrac{(-3)^3 a^3 b^9 a^{25} b^{10}}{(3)^2 a^4 b^8 c^2} = \dfrac{-27\, a^{28} b^{19}}{9\, a^4 b^8 c^2}$

$= -\dfrac{3\, a^{24} b^{11}}{1\, c^2} = \boxed{-\dfrac{3a^{24}b^{11}}{c^2}}$

Ex: $\dfrac{a^8}{a^{15}} = \boxed{\dfrac{1}{a^7}}$

Also $\dfrac{a^8}{a^{15}} = a^{8-15} = a^{-7}$ This is defined to be $\dfrac{1}{a^7}$

(Negative exponents are covered in 5.3)