5.3: Definitions of b^0 and b^{-n} (Negative Exponents)

Note Title 2/20/2017

Review Problems:

(a) $(-x)^5 = (-1x)^5 = (-1)^5 x^5 = -1x^5 = \boxed{-x^5}$

(b) $(-x^2y^3z)^6$

$= (-1x^2y^3z)^6 = (-1)^6 x^{12} y^{18} z^6 = 1x^{12}y^{18}z^6$

$= \boxed{x^{12}y^{18}z^6}$

On to Section 5.3

What does 2^0 mean?

$$\frac{2^3}{2^3} = \frac{8}{8} = 1$$

Also $$\frac{2^3}{2^3} = 2^{3-3} = 2^0$$

must be equal

So, $2^0 = 1$.

Similarly, $5^0 = 1$, $3^0 = 1$, $(-17)^0 = 1$.

Note: $2(0) = 0$ and $2^0 = 1$

Prop. of Exponents

$$\frac{x^a}{x^b} = x^{a-b}$$

Important Fact:

$$x^0 = 1 \quad \text{as long as } x \neq 0. \quad \bigstar$$

(0^0 is considered undefined)

Ex: $21^0 = (21^0) = 1$

$(-37)^0 = 1$

$(5x^2y^3c^8)^0 = 1$

(as long as $x \neq 0, y \neq 0, c \neq 0$)

Definition of a negative exponent: $\bigstar$

$$x^{-n} = \frac{1}{x^n}$$

Note: $x^{-n} = \frac{x^{-n}}{1} = \frac{1}{x^n}$

$\frac{1}{x^{-n}} = \frac{x^n}{1} = x^n$. Why: $\frac{1}{x^{-n}} = \frac{1}{\frac{1}{x^n}}$

$= 1 \cdot \frac{x^n}{1} = x^n$

$\bigstar$ $\frac{x^{-n}}{1} = \frac{1}{x^n}$ $\bigstar$

$\frac{1}{x^{-n}} = \frac{x^n}{1}$

Example: $5^{-2} = \frac{5^{-2}}{1} = \frac{1}{5^2} = \boxed{\frac{1}{25}}$

In all these examples, we will simplify, and write answers with positive exponents only.

Ex: $-2^{-4} = -\frac{2^{-4}}{1} = -\frac{1}{2^4} = \boxed{-\frac{1}{16}}$

Ex: $(-3)^{-4} = \frac{(-3)^{-4}}{1} = \frac{1}{(-3)^4} = \boxed{\frac{1}{81}}$

Ex: $\left(\frac{3}{5}\right)^{-2} = \frac{3^{-2}}{5^{-2}} = \frac{5^2}{3^2} = \boxed{\frac{25}{9}}$

OR $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^{2} = \frac{5^2}{3^2} = \boxed{\frac{25}{9}}$

Ex: $\left(-\frac{2}{3}\right)^{-4} = \left(-1\left(\frac{2}{3}\right)\right)^{-4} = (-1)^{-4}\left(\frac{2}{3}\right)^{-4} = \frac{1}{(-1)^4} \cdot \frac{2^{-4}}{3^{-4}}$

$= \frac{1}{1} \cdot \frac{3^4}{2^4} = \boxed{\frac{81}{16}}$

OR $\left(-\frac{2}{3}\right)^{-4} = \left(\frac{-2}{3}\right)^{-4} = \frac{(-2)^{-4}}{3^{-4}} = \frac{3^4}{(-2)^4} = \boxed{\frac{81}{16}}$

Ex: $(2x)^{-3} = \frac{(2x)^{-3}}{1} = \frac{2^{-3}x^{-3}}{1}$

$= \frac{1}{2^3 x^3} = \boxed{\frac{1}{8x^3}}$

Compare!

Ex: $\frac{1}{6^{-2}} = \frac{6^2}{1} = \frac{36}{1} = \boxed{36}$

Ex: $3x^{-4} = \frac{3x^{-4}}{1} = \boxed{\frac{3}{x^4}}$

Ex: $3^{-1} + 3^{-2} = \frac{3^{-1}}{1} + \frac{3^{-2}}{1} = \frac{1}{3^1} + \frac{1}{3^2}$

$= \frac{1}{3} + \frac{1}{9} = \frac{1}{3} \cdot \frac{3}{3} + \frac{1}{9}$

$= \frac{3}{9} + \frac{1}{9} = \boxed{\frac{4}{9}}$

Ex: $\frac{x^{-4}y^{-3}}{x^5 y^{-8}} = \frac{1 \; y^8}{x^4 y^3 x^5} = \frac{y^8}{x^9 y^3} = \boxed{\frac{y^5}{x^9}}$

Ex: $\left(\frac{2x}{y^4 z^{-1}}\right)^{-3} = \frac{2^{-3} x^{-6}}{y^{-12} z^3} = \frac{1\, y^{12}}{2^3 x^6 z^3} = \boxed{\frac{y^{12}}{8 x^6 z^3}}$

Ex: $\frac{-3x^{-5}\left(x^{-1}y^3 z^{-2}\right)^{-4}}{(-3x^2 y^4)^{-2}(2x y^{-3} z^4)} = \frac{-3x^{-5} x^4 y^{-12} z^8}{(-3)^{-2} x^{-4} y^{-8} 2^4 x^1 y^{-12} z^4}$

$= \frac{-3}{x^5} \cdot \frac{x^4}{y^{12}} \cdot \frac{z^8 (-3)^2 x^4 y^8}{2^4 x^1} \cdot \frac{y^{12}}{z^4}$

$= \frac{-3 \cdot 9\, x^8 y^{20} z^8}{2^4 x^9 y^{12} z^4} = \boxed{\frac{-27 y^8 z^4}{16 x}} = \boxed{-\frac{27 y^8 z^4}{16 x}}$

HW 6: $(x^m)^2$ 5.2 # 75

$x^{m \cdot 2} = x^{2m}$