# Revisiting 5.3

Note Title 2/22/2017

Homework Qs 103 and 105 ask you to choose the exponent that makes the equations true.

Ex: $\dfrac{a^{\square} a^{12}}{a^{10}} = a^7$

$\dfrac{a^{\square} a^{2}}{1} = a^7$

$a^{\square} a^{2} = a^7$

The missing exponent must be 5.

Check: $\dfrac{a^5 a^{12}}{a^{10}} = a^7$

$\dfrac{a^{17}}{a^{10}} = a^7$

$a^7 = a^7$ ✓ True

Ex: $\dfrac{a^{\square} a^{-2}}{a^{-6} a} = a^{-3}$

$\dfrac{a^{\square} a^{6}}{a^{2} a} = \dfrac{1}{a^3}$

$\dfrac{a^{\square} a^{6}}{a^{3}} = \dfrac{1}{a^3}$

$\dfrac{a^{\square} a^{3}}{1} = \dfrac{1}{a^3}$

Is the missing exponent 3?

Try it: $\dfrac{a^{\boxed{3}} a^{3}}{1} = \dfrac{1}{a^3}$

Is the missing exponent −4?

Try it: $\dfrac{a^{\boxed{-4}} a^{3}}{1} = \dfrac{1}{a^3}$

$\dfrac{1}{a} = \dfrac{a^3}{a^4} \neq \dfrac{1}{a^3}$

Is the missing exponent −6?

Try it: $\dfrac{a^{\boxed{-6}} a^{3}}{1} = \dfrac{1}{a^3}$

$\dfrac{a^3}{a^6} = \dfrac{1}{a^3}$

$\dfrac{1}{a^3} = \dfrac{1}{a^3}$ True!

The missing exponent is −6.

# Homework Qs

5.3 # 87

$$\frac{(-4c^{12}d^{7})^{2}}{(5c^{-3}d^{10})^{-1}} = \frac{(-4)^{2}c^{24}d^{14}}{5^{-1}c^{3}d^{-10}}$$

$$= \frac{16c^{24}d^{14} \cdot 5^{1}d^{10}}{c^{3}} = \frac{80c^{21}d^{24}}{1}$$

$$\begin{array}{r} \overset{3}{1}6 \\ 5 \\ \hline 80 \end{array}$$

$$= \boxed{80c^{21}d^{24}}$$

# 5.5: Addition and Subtraction of Polynomials

Terminology:

A polynomial expression, or polynomial, is a sum of finitely many terms composed of coefficients and variables to positive integer exponents.

Examples of polynomials:

$3y^2 + 5x^2 - 7xy$

$2x^4 - 7x^3 + 6x - 2$

$7 - 8x^2$

$\frac{1}{2}x^4 - 7x^2 + \frac{3}{5}x - \frac{1}{4}$

Not polynomials:

$2x^{-4} + 5x^{-1}$

$\frac{3}{7x^2} - \frac{8}{5x}$

$3x^{\frac{1}{2}} - x^{1/3}$

(same as $3\sqrt{x} - \sqrt[3]{x}$)

Degree of a polynomial:

* If it has 1 variable, the degree is the largest exponent.
* If it has 2 or more variables, the degree is the largest total exponent in a term.

Ex: The degree of $-2x^4 + x^3 - 7x^2 + 5x + 2$ is 4.
(this is a 4th degree polynomial)

Ex: What is the degree of $2x - 3x^7$? 7
(it's a 7th degree polynomial).

Ex: What is the degree?

$$\underbrace{2x^2y^3}_{\text{total exponent } 5} + \underbrace{4x^5y^4}_{\text{total exponent } 9} - \underbrace{7xy^5}_{\text{total exponent } 6} - \underbrace{8y^4}_{\text{total exponent } 4}$$

Biggest

$\boxed{\text{The degree is 9.}}$

Special names for certain polynomials

Polynomial with 3 terms: trinomial

Polynomial with 2 terms: binomial

Polynomial with 1 term: monomial

The leading term is the term with the largest exponent on the variable.

The leading coefficient is the coefficient of the leading term.

Ex: $3x - x^4$ is a 4th degree binomial.

Leading term: $-x^4$

Leading coefficient: $-1$ (coefficient of $-x^4$)

Ex: $3y^2 - 8y + 12$ is a 2nd degree trinomial (degree 2)

Leading term: $3y^2$

Leading coefficient: 3

# Adding and subtracting polynomials

We already know how to do this. We combine like terms.

**Ex:** $(5a+3b)+(3a-7b)$ Simplify.

$$5a+3b+3a-7b$$

$$= \boxed{8a-4b}$$

**Ex:** $(4x^2+8x-6)-(-6x^2-x+9)$

$$= 4x^2+8x-6-1(-6x^2-x+9)$$

$$= 4x^2+8x-6+6x^2+x-9$$

$$= \boxed{10x^2+9x-15}$$

# 5.6: Multiplication of Polynomials

We already know how to multiply monomials:

Ex: $(-5x^4)(3x^5)$

$= \boxed{-15x^9}$

Ex: $-a^4bc^2(-3ac^4)(5b^3)$

$= \boxed{15a^5b^4c^6}$

Multiplying a polynomial by a monomial:

we distribute.

Ex: Simplify.

$3x^3(2x^2 - 8x + 7)$

$= 3x^3(2x^2) + 3x^3(-8x) + 3x^3(7)$

$= \boxed{6x^5 - 24x^4 + 21x^3}$

Ex: $-5x^2y^3(-xy^4 + 8xy - 3x^3y^5 - 2y^2)$

$= -5x^2y^3(-xy^4) - 5x^2y^3(8xy) - 5x^2y^3(-3x^3y^5) - 5x^2y^3(-2y^2)$

$= \boxed{5x^3y^7 - 40x^3y^4 + 15x^5y^8 + 10x^2y^5}$

## Multiplying 2 polynomials that have 2 or more terms

Recall: Distributive property:

$$c(a+b) = ca + cb$$

$$(a+b)(c) = ac + bc$$

Example: multiply.

$$(\underbrace{3x}_{a} + \underbrace{5}_{b})\underbrace{(x-7)}_{c}$$

$$= \underbrace{3x(x-7)}_{ac} + \underbrace{5(x-7)}_{bc}$$

$$= 3x^2 - 21x + 5x - 35$$

$$= \boxed{3x^2 - 16x - 35}$$

Ex: $(2x-1)(6x-5)$

$$= 2x(6x-5) - 1(6x-5)$$

$$= 12x^2 - 10x - 6x + 5$$

$$= \boxed{12x^2 - 16x + 5}$$

OR

$$(2x-1)(6x) + (2x-1)(-5)$$

$$12x^2 - 6x - 10x + 5$$

$$12x^2 - 16x + 5$$

Ex: $(x+2y)(-5x^2 - xy + 8y^3)$

$$= x(-5x^2 - xy + 8y^3) + 2y(-5x^2 - xy + 8y^3)$$

$$= -5x^3 - x^2y + 8xy^3 - 10x^2y - 2xy^2 + 16y^4$$

$$= \boxed{-5x^3 - 11x^2y + 8xy^3 - 2xy^2 + 16y^4}$$

Ex: $(x^2-7x-1)(2x^2-5x-6)$

$= x^2(2x^2-5x-6) - 7x(2x^2-5x-6) - 1(2x^2-5x-6)$

$$
\begin{aligned}
= 2x^4 - 5x^3 &- 6x^2 \\
- 14x^3 &+ 35x^2 + 42x \\
&- 2x^2 + 5x + 6
\end{aligned}
$$

$= \boxed{2x^4 - 19x^3 + 27x^2 + 47x + 6}$

The FOIL method (for multiplying 2 binomials)
(a mnemonic device)

F : First

O : Outer

I : Inner

L : Last

Ex:
First Last

$(2x+3)(5x-4)$

Inner Outer

$= 10x^2 - 8x + 15x - 12$

$= \boxed{10x^2 + 7x - 12}$

Ex:
F L

$(-3x+y)(12x-2y)$

I O

$= -36x^2 + \underline{6xy} + \underline{12xy} - 2y^2$

$= \boxed{-36x^2 + 18xy - 2y^2}$

Ex: $(y-4)(y+4)$

$= y^2 + \underline{4y} - \underline{4y} - 16$

add up to 0

$= \boxed{y^2 - 16}$

## Difference of two squares Pattern

$$(a+b)(a-b) = a^2 - b^2$$

Check: $a^2 - \cancel{ab} + \cancel{ab} - b^2$
$= a^2 - b^2$

Ex: $(2x+5)(2x-5)$
$= 2x(2x-5) + 5(2x-5)$
$= 4x^2 - 10x + 10x - 25$
$= \boxed{4x^2 - 25}$

Perfect Squares: $(3x+1)^2$
$= (3x+1)(3x+1)$
$= 9x^2 + 3x + 3x + 1$
$= \boxed{9x^2 + 6x + 1}$

Ex: $(3x+1)^3$
$= (3x+1)(3x+1)(3x+1)$
$= (9x^2 + 3x + 3x + 1)(3x+1)$
$= (9x^2 + 6x + 1)(3x+1)$
$= 9x^2(3x+1) + 6x(3x+1) + 1(3x+1)$
$= 27x^3 + 9x^2 + 18x^2 + 6x + 3x + 1$
$= 27x^3 + 27x^2 + 9x + 1$