5.7: Dividing Polynomials

Note Title 2/27/2017

Dividing a polynomial by a monomial

Ex: Divide.

$$\frac{6x^4 - 8x^3 + 12x^2 - 9}{2x^2}$$

Split into separate fractions:

$$\frac{6x^4 - 8x^3 + 12x^2 - 9}{2x^2} = \frac{6x^4}{2x^2} - \frac{8x^3}{2x^2} + \frac{12x^2}{2x^2} - \frac{9}{2x^2}$$

$$= \frac{3x^2}{1} - \frac{4x}{1} + \frac{6}{1} - \frac{9}{2x^2}$$

$$= \boxed{3x^2 - 4x + 6 - \frac{9}{2x^2}}$$

Note:

$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$$

Ex: Divide.

$$\frac{2x^4y^3 + 8x^3y^3 - 4y^2}{2xy^2}$$

$$\frac{2x^4y^3 + 8x^3y^3 - 4y^2}{2xy^2} = \frac{2x^4y^3}{2xy^2} + \frac{8x^3y^3}{2xy^2} - \frac{4y^2}{2xy^2}$$

$$= \frac{x^3y}{1} + \frac{4x^2y}{1} - \frac{2}{x}$$

$$= \boxed{x^3y + 4x^2y - \frac{2}{x}}$$

Dividing a polynomial by a polynomial

Example: Divide $\dfrac{5x^2 - 7x + 6}{x + 3}$

Could write as separate fractions: $\dfrac{5x^2}{x+3} - \dfrac{7x}{x+3} + \dfrac{6}{x+3}$

Cannot reduce any of these, because x is not connected to rest of denominator by multiplication.

So, this is a dead end (though sometimes is useful ... in calculus)

We need <u>polynomial long division</u>.

Review: Long Division

Example: Divide $\frac{379}{12}$.

$$
\begin{array}{r}
31 \\
12\overline{)379} \\
-\,36 \\
\hline
19 \\
-\,12 \\
\hline
7
\end{array}
$$

Write answer:

$$\frac{379}{12} = \boxed{31 + \frac{7}{12}}$$ (yes, it's usually written $3\frac{7}{12}$)

Check answer: $12(31) + 7 = 372 + 7 = 379$ ✓

$$
\begin{array}{r}
31 \\
\times\ 12 \\
\hline
62 \\
31 \\
\hline
372
\end{array}
$$

Terminology:

$$
\begin{array}{r}
\text{Quotient} \\
\text{Divisor}\ \overline{)\ \text{Dividend}} \\
\vdots \\
\hline
\text{Remainder}
\end{array}
$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$$ [multiply both sides by Divisor]

Example: Divide. $\dfrac{3x^2+7x+9}{x+2}$

$x(?) = 3x^2$ Answer: $3x$

$x(?) = x$ Answer: 1

$\frac{3x^2}{x}$ $\quad$ $\frac{x}{x}$

$$x+2 \;\overline{)\; 3x^2+7x+9}$$

$$3x+1$$

$$-(3x^2+6x) \quad \leftarrow 3x(x+2)$$

$$x+9$$

$$-(x+2) \quad \leftarrow 1(x+2) = x+2$$

$$7$$

Write answer.

$$\frac{3x^2+7x+9}{x+2} = \boxed{3x+1+\frac{7}{x+2}}$$

Note: When $x=10$, this is identical to the previous numerical example

Check answer:

$$(x+2)(3x+1)+7$$
$$= 3x^2+1x+6x+2+7$$
$$= 3x^2+7x+9 \quad \checkmark \text{ ok} \leftarrow \text{same as original numerator/dividend}$$

Example: Divide. $\dfrac{x^3-6x^2+x+14}{x-5}$

$\frac{x^3}{x}$ $\frac{-x^2}{x}$ $\frac{-4x}{x}$

$$\begin{array}{r|l} x-5 & x^3-6x^2+x+14 \\ & \ominus\ (x^3-5x^2) \quad \leftarrow x^2(x-5)=x^3-5x^2 \\ \hline & -x^2+x \\ & \ominus\ (-x^2+5x) \quad \leftarrow -x(x-5)=-x^2+5x \\ \hline & -4x+14 \\ & \ominus\ (-4x+20) \\ \hline & -6 \end{array}$$

x^2-x-4 ← top row

Write answer:

$$\frac{x^3-6x^2+x+14}{x-5} = x^2-x-4+\frac{-6}{x-5} = x^2-x-4-\frac{6}{x-5}$$

both are correct

Check answer:

$$(x-5)(x^2-x-4)-6$$

$$= x^3-x^2-4x$$
$$-5x^2+5x+20-6$$
$$= x^3-6x^2+x+14 \quad \checkmark \text{ok}$$

Example: Divide. $\dfrac{8x^3 - 10x^2 + x - 2}{2x-1}$

$\frac{8x^3}{2x}$ $\frac{-6x^2}{2x}$ $\frac{-2x}{2x}$

$$\begin{array}{r} 4x^2 - 3x - 1 \\ 2x-1 \overline{)\,8x^3 - 10x^2 + x - 2} \\ \ominus\ (8x^3 \ominus 4x^2) \\ \hline -6x^2 + x \\ \ominus\ (\ominus 6x^2 \oplus 3x) \\ \hline -2x - 2 \\ \oplus\ (\oplus 2x \ominus 1) \\ \hline -3 \end{array}$$

$4x^2(2x-1) = 8x^3 - 4x^2$

$-3x(2x-1) = -3x^2 + 3x$

$-1(2x-1) = -2x + 1$

Write answer:

$$\frac{8x^3 - 10x^2 + x - 2}{2x-1} = 4x^2 - 3x - 1 + \frac{-3}{2x-1}$$

both are correct

$$= 4x^2 - 3x - 1 - \frac{3}{2x-1}$$

Check your answer:

$$(2x-1)(4x^2 - 3x - 1) - 3$$

$$= 8x^3 - 6x^2 - 2x - 4x^2 + 3x + 1 - 3$$

$$= 8x^3 - 10x^2 + x - 2 \quad \checkmark \text{ok}$$

Example. Divide. $\dfrac{2x^3 - 77x - 12}{x+6}$

Important: You must put "placeholders" (zero-coefficient terms) in place of missing powers of x. In this example, we put $0x^2$ for the x^2 term.

$\frac{2x^3}{x}$ $\quad$ $\frac{-12x^2}{x}$ $\quad$ $\frac{-5x}{x}$

$$\begin{array}{r} 2x^2 - 12x - 5 \\ x+6 \overline{)\,2x^3 + 0x^2 - 77x - 12} \\ \pm(-2x^3 \mp 12x^2) \\ \hline -12x^2 - 77x \\ \mp(\pm 12x^2 \mp 72x) \\ \hline -5x - 12 \\ \pm(\mp 5x \mp 30) \\ \hline 18 \end{array}$$

$2x^2(x+6) = 2x^3 + 12x^2$

$-6x(x+6) = -6x^2 - 36x$

$-5(x+6) = -5x - 30$

Write answer:

$$\frac{2x^3 - 77x - 12}{x+6} = \boxed{2x^2 - 12x - 5 + \frac{18}{x+6}}$$

Check answer:

$$\begin{aligned} &(x+6)(2x^2 - 12x - 5) + 18 \\ &= 2x^3 - 12x^2 - 5x \\ &\quad + 12x^2 - 72x - 30 + 18 \\ &= 2x^3 - 77x - 12 \quad \checkmark \text{ok} \end{aligned}$$