5.7: Long Division (cont'd)

Note Title 3/1/2017

5.7 # 58

Divide. $\dfrac{2x^4+6x+4}{x^2+2}$

$$
\begin{array}{r}
2x^2 - 4 \\
x^2+0x+2 \;\overline{\big)\, 2x^4+0x^3+0x^2+6x+4} \\
\oplus\; -(2x^4 - 0x^3 - 4x^2) \\
\hline
-4x^2+6x+4 \\
\oplus\; (+4x^2 + 0x + 8) \\
\hline
6x+12
\end{array}
$$

In general: Degree of the remainder will be 1 lesss than the degree of the divisor.

So, if divisor is 2^{nd} degree, I expect remainder to be 1^{st} degree (an x term and a constant term)

If divisor is 1^{st} degree, we expect remainder to be a constant.

(But sometimes the x term and/or the constant term can 0 out)

write answer: $\dfrac{2x^4+6x+4}{x^2+2} = 2x^2-4+\dfrac{6x+12}{x^2+2}$

Check: $(x^2+2)(2x^2-4)+6x+12 = 2x^4 - \cancel{4x^2} + \cancel{4x^2} - 8 + 6x + 12$
$= 2x^4 + 6x + 4$ ✓

Example: Divide $\dfrac{10x^3+21x^2-5}{2x^2+5x+2}$

$$
\begin{array}{r|l}
 & 5x \quad -2 \\
\hline
2x^2+5x+2 & 10x^3 + 21x^2 + 0x - 5 \\
+ & -(10x^3 + 25x^2 + 10x) \\
\hline
 & -4x^2 - 10x - 5 \\
+ & -(-4x^2 - 10x - 4) \\
\hline
 & -1
\end{array}
$$

$$\frac{10x^3+21x^2-5}{2x^2+5x+2} = \boxed{5x-2+\frac{-1}{2x^2+5x+2}}$$

$$= \boxed{5x-2-\frac{1}{2x^2+5x+2}}$$

both correct

Chapter 6: Factoring Polynomials

$(x+3)(x+4)$ ← Factored Form

$= x^2 + 7x + 12$ ← Unfactored Form

In this chapter, we'll start with polynomials in unfactored form and break them down into a product of factors.

Ex: $2(3) = 6$, so 2 and 3 are factors of 6.

6.1: The Greatest Common Factor and Factoring by Grouping

To find the greatest common factor (GCF):

* Find the largest number that is a factor of all the coefficients.
* Find the largest power of each variable that is a factor of all the terms. (if a variable appears in all the terms, choose the smallest exponent)
* Multiply the numerical part and the variable part together to get the GCF.

Example: Factor out the GCF.

$15x^4 - 6x^2$

GCF: $3x^2$

$= 3x^2\left(\frac{15x^4}{3x^2} - \frac{6x^2}{3x^2}\right)$

$= \boxed{3x^2(5x^2 - 2)}$

Check by multiplying:

$3x^2(5x^2 - 2)$

$= 15x^4 - 6x^2$ ✓

Example: Factor completely. (Factor.)

$48x^3 - 32x$

~~G~~CF: $8x$

8x is a common factor but not the greatest common factor

$= 8x(\underbrace{6x^2 - 4}_{\text{not done yet! These have a common factor of 2}})$

Check: $8x(6x^2-4)$
$= 48x^3 - 32x$ ✓

$= 8x(2)(3x^2-2)$

$= \boxed{16x(3x^2-2)}$

Check: $16x(3x^2-2)$
$= 48x^3 - 32x$ ✓

$\begin{array}{r} 16 \\ \times 3 \\ \hline 48 \end{array}$

OR, notice in the beginning that the GCF is $16x$

$48x^3 - 32x$

GCF: $16x$

$= \boxed{16x(3x^2 - 2)}$

Ex: Factor.

$24x^3 + 18x^2 - 6$

GCF: 6

$= \boxed{6(4x^3 + 3x^2 - 1)}$

Ex: Factor.

$-30x^4y^3z + 12x^3y^6 - 48x^4y^2z$

GCF: $6x^3y^2$

$= 6x^3y^2\left(\frac{-30x^4y^3z}{6x^3y^2} + \frac{12x^3y^6}{6x^3y^2} - \frac{48x^4y^2z}{6x^3y^2}\right)$

$= \boxed{6x^3y^2(-5xyz + 2y^4 - 8xz)}$

Check it!

Factoring by Grouping

Example: Factor.

GCF: 1

$$3x^3 - 7x^2 + 15x - 35$$
$$= (3x^3 - 7x^2) + (15x - 35)$$
$$= x^2(3x - 7) + 5(3x - 7)$$
$$= \boxed{(3x-7)(x^2+5)}$$

Check it: $3x^3 + 15x - 7x^2 - 35$ ✓ ok

Also correct: $\boxed{(x^2+5)(3x-7)}$

Ex.: Factor.

$$4x^3 - 3x^2 - 36x + 27$$
$$= (4x^3 - 3x^2) + (-36x + 27)$$
$$= x^2(4x - 3) - 9(4x - 3)$$
$$= \boxed{(4x-3)(x^2-9)}$$

or $(4x^3 - 3x^2) + (-36x + 27)$

$= x^2(4x-3) + 9(-4x + 3)$

(parentheses don't match)

$= x^2(4x-3) - 9(4x-3)$

Check: $4x^3 - 36x - 3x^2 + 27$ ✓

final answer $\Rightarrow$ $= \boxed{(4x-3)(x+3)(x-3)}$ ← we haven't done this factorization yet... we'll get to it after spring break

6.1 #51)

$$21x(x+3) + 7x^2(x+3)$$

$$= (x+3)(21x+7x^2)$$

$$= (x+3)(7x)(3+x)$$

$$= (x+3)(7x)(x+3)$$

$$= 7x(x+3)(x+3)$$

$$= \boxed{7x(x+3)^2}$$

Ex: $-16x^2y - 4x^2 + 24xy + 6x$ GCF: $2x$

$$= 2x(-8xy - 2x + 12y + 3)$$

$$= 2x[2x(-4y-1) + 3(4y+1)]$$

$$= 2x[-2x(4y+1) + 3(4y+1)]$$

$$= 2x[(4y+1)(-2x+3)]$$

$$= \boxed{2x(4y+1)(-2x+3)}$$

6.2: Factoring Trinomials

Example: Factor.

F First
O Outer
I Inner
L Last

$x^2 + 6x + 8$

$$\boxed{(x+2)(x+4)}$$

Check: $x^2 + 4x + 2x + 8$
$= x^2 + 6x + 8$ ✓

(+) same signs

Ex: $x^2 - 10x + 24$

$$\boxed{(x-4)(x-6)}$$

Check: $x^2 - 6x - 4x + 24$
$= x^2 - 10x + 24$ ✓

same signs, I want a sum of 10

24
$\wedge$
1·24
2·12
3·8
4·6

(−) signs are opposite

Ex: $x^2 + 10x - 24$

$$\boxed{(x-2)(x+12)}$$

Check: $x^2 + 12x - 2x - 24$
$= x^2 + 10x - 24$

signs are opposite, so I want a difference of 10

24
$\wedge$
1·24
2·12
3·8
4·6

(−) signs are opposite; so I want a difference of 10

Ex: $x^2 - 10x - 24$

$$\boxed{(x+2)(x-12)}$$

Check: $x^2 - 12x + 2x - 24$
$= x^2 - 10x - 24$ ✓

24
$\wedge$
1·24
2·12
3·8
4·6

Ex: $x^2 + 10x + 24$ (+) same signs
want sum of 10

$$\boxed{(x+4)(x+6)}$$

24

1 · 24
2 · 12
3 · 8
4 · 6

Check: $x^2 + 6x + 4x + 24$
$= x^2 + 10x + 24$