

# 6.3: Factoring Trinomials (leading coefficient > 1)

Note Title

(cont'd)

3/20/2017

HW Qs

6.3 #1b

Which of these is the complete factorization of  $6x^2 - 4x - 10$ ?

Choices are  $(3x-5)(2x+2)$   
 $2(3x-5)(x+1)$

1st, Do these choices multiply out to give us  $6x^2 - 4x - 10$ ?

check:  $(3x-5)(2x+2)$   
 $= 6x^2 + 6x - 10x - 10$   
 $= 6x^2 - 4x - 10 \quad \checkmark$

Check the other:  $2(3x-5)(x+1)$

Note: When you have 3 numbers multiplied together, you choose 2 of them to multiply 1st.

$$\begin{array}{lll} 2(5)(3) & \text{or} & 2(5)(3) \quad \text{or} \quad 2(5)(3) \\ = 10(3) & & = 2(3)(5) \quad = 2(15) \\ = 30 & & = 6(5) \\ & & = 30 \end{array}$$

Note that  $2(5+3)$  same result as  
 $= 2(5) + 2(3)$   $2(5+3)$   
 $= 10 + 6$   $= 2(8)$   
 $= 16$   $= 16$

Back to our problem: Check the 2nd choice:

$$\begin{aligned} & 2(3x-5)(x+1) \\ & = (6x-10)(x+1) \\ & = 6x^2 + 6x - 10x - 10 = 6x^2 - 4x - 10 \quad \checkmark \end{aligned}$$

see next page

#1 b cont'd:

So both of the choices multiply out to equal the original.

So  $(3x-5)(2x+2)$  is not completely factored, because we can factor 2 out of  $2x+2$ .

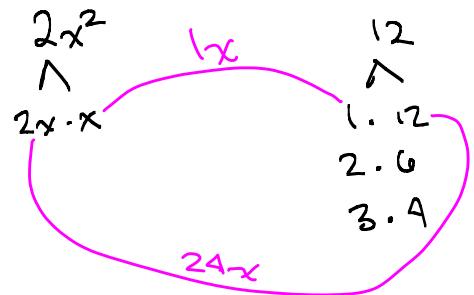
$$\begin{aligned} & (3x-5)(2x+2) \\ &= (3x-5)(2)(x+1) \\ &= \boxed{2(3x-5)(x+1)} \quad \text{is the } \underline{\text{complete}} \text{ factorization} \end{aligned}$$

More examples from 6.3 (factoring trinomials)

Example:  $2x^2 - 25x + 12$  (+) same signs want sum of  $25x$  for middle term Factor.

$$\boxed{(2x - 1)(x - 12)}$$

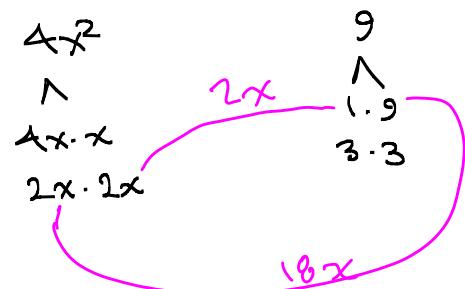
Check:  $2x^2 - 24x - 1x + 12$   
 $2x^2 - 25x + 12 \checkmark$



Ex.:  $4x^2 + 16x - 9$  (-) signs are opposite want a difference of  $16x$  for middle term

$$\boxed{(2x - 1)(2x + 9)}$$

Check:  $4x^2 + 8x - 2x - 9$   
 $= 4x^2 + 16x - 9$



Try these:-

1)  $6x^2 + 19x + 14$

2)  $7x^2 - 10x + 3$

3)  $10x^2 + 49x - 5$

4)  $2x^2 - 11xy + 5y^2$

Ex.  $18x^3 + 15x^2 - 18x$

$$= 3x(6x^2 + 5x - 6)$$

$$= \boxed{3x (2x + 3)(3x - 2)}$$

signs are opposite  
 want a difference  
 of  $5x$

Check:  $(2x+3)(3x-2)$

$$= 6x^2 - 4x + 9x - 6$$

$$= 6x^2 + 5x - 6 \quad \checkmark \text{OK}$$

$$\begin{array}{r}
 6x^2 \\
 \diagdown \\
 6x \cdot x \\
 2x \cdot 3x
 \end{array}
 \quad
 \begin{array}{r}
 6 \\
 \diagup \\
 1 \cdot 6 \\
 2 \cdot 3
 \end{array}$$

$6x^2$   
 $\Delta x$   
 $9x$

Ex.  $-6x^2 + 11x - 4$

$$= -1(6x^2 - 11x + 4)$$

$$= -1(2x - 1)(3x - 4)$$

(+) same signs  
 want sum of  $11x$

Check:

$$(2x-1)(3x-4)$$

$$= 6x^2 - 8x - 3x + 4$$

$$= 6x^2 - 11x + 4$$

$$\begin{array}{r}
 6x^2 \\
 \diagdown \\
 6x \cdot x \\
 2x \cdot 3x
 \end{array}
 \quad
 \begin{array}{r}
 4 \\
 \diagup \\
 1 \cdot 4 \\
 2 \cdot 2
 \end{array}$$

$6x^2$   
 $3x$   
 $8x$

Final answer:

$-(2x - 1)(3x - 4)$

## 6.4: Factoring trinomials (leading coefficient >1) using the AC method:

Work the 6.4 HW the same way  
as the 6.3 homework.

## 6.5: Difference of Two Squares and Perfect Square Trinomials

Recall: 
$$\begin{aligned} (a+b)(a-b) &= a^2 - ab + ab - b^2 \\ &= a^2 - b^2 \end{aligned}$$

Difference of Two Squares Factorization

$$a^2 - b^2 = (a+b)(a-b)$$

Ex: Factor.

$$\begin{aligned} x^2 - 25 &= x^2 - 5^2 \\ &= \boxed{(x+5)(x-5)} \end{aligned}$$

Check:  $x^2 - 5x + 5x - 25$   
 $= x^2 - 25 \checkmark$

Perfect Squares:

$$\begin{array}{ll} 1^2 = 1 & 8^2 = 64 \\ 2^2 = 4 & 9^2 = 81 \\ 3^2 = 9 & 10^2 = 100 \\ 4^2 = 16 & 11^2 = 121 \\ 5^2 = 25 & 12^2 = 144 \\ 6^2 = 36 & \\ 7^2 = 49 & \end{array}$$

Example: Factor.

$$\begin{aligned} 4x^2 - 9 &= (2x)^2 - (3)^2 \\ &= \boxed{(2x+3)(2x-3)} \end{aligned}$$

Check:  $4x^2 - 6x + 6x - 9 = 4x^2 - 9 \checkmark$

$$\text{or } \boxed{(2x-3)(2x+3)}$$

Ex.  $100y^2 - 81z^2$

$$\begin{aligned} & (10y)^2 - (9z)^2 \\ & \boxed{(10y + 9z)(10y - 9z)} \end{aligned}$$

Check it!

Ex.  $32x^2 - 50y^2$

$$\begin{aligned} & = 2(16x^2 - 25y^2) \\ & = 2((4x)^2 - (5y)^2) \\ & = \boxed{2(4x - 5y)(4x + 5y)} \end{aligned}$$

$$\begin{aligned} \text{Check: } & (4x - 5y)(4x + 5y) \\ & = 16x^2 + 20xy - 20xy - 25y^2 \\ & = 16x^2 - 25y^2 \quad \checkmark \end{aligned}$$

Ex.  $x^5 - x$

$$\begin{aligned} & x(x^4 - 1) \\ & x((x^2)^2 - (1^2)^2) \\ & x(x^2 - 1)(x^2 + 1) \\ & x(x+1)(x-1)(x^2 + 1) \\ & \boxed{x(x+1)(x-1)(x^2 + 1)} \end{aligned}$$

Note:  $x^2 + 1$  is prime

Ex.  $x^2 + 49$

$$\begin{aligned} \text{Try: } & (x+7)(x+7) \\ & x^2 + 7x + 7x + 49 \\ & = x^2 + 14x + 49 \quad \text{No!} \end{aligned}$$

$$\begin{aligned} \text{Try: } & (x+7)(x-7) \\ & x^2 - 7x + 7x - 49 \\ & x^2 - 49 \quad \text{No!} \end{aligned}$$

$x^2 + 49$  is Prime.

Important Fact:  $a^2 + b^2$  is always prime.  
 $x^2 + \text{positive}$  is always prime.



Note:-

$$x^2 + 49$$

Rewrite:

$$x^2 + 0x + 49$$

It's impossible to get a  
sum of 0.

(+) same signs  
want sum

of 0x

$$\begin{array}{ccc} x^2 & & 49 \\ \swarrow & & \nearrow \\ x \cdot x & 7x & 7 \cdot 7 \\ \text{sum is } 14x \end{array}$$