6.3: Factoring Trinomials (leading coefficient > 1) (cont'd)

Note Title 3/20/2017

HW Qs

6.3 #16 Which of these is the <u>complete</u> factorization of $6x^2-4x-10$?

Choices are $(3x-5)(2x+2)$

$2(3x-5)(x+1)$

1st, Do these choices multiply out to give us $6x^2-4x-10$?

check: $(3x-5)(2x+2)$

$= 6x^2+6x-10x-10$

$= 6x^2-4x-10$ ✓

Check the other: $2(3x-5)(x+1)$

<u>Note</u>: When you have 3 numbers multiplied together, you choose 2 of them to multiply 1st.

$2(5)(3)$
$= 10(3)$
$= 30$

or $2(5)(3)$
$= 2(3)(5)$
$= 6(5)$
$= 30$

or $2(5)(3)$
$= 2(15)$
$= 30$

Note that $2(5+3)$
$= 2(5)+2(3)$
$= 10+6$
$= 16$

same result as $2(5+3)$
$= 2(8)$
$= 16$

Back to our problem: Check the 2nd choice:

$2(3x-5)(x+1)$

$= (6x-10)(x+1)$

$= 6x^2+6x-10x-10 = 6x^2-4x-10$ ✓ ok

see next page

#16 cont'd:

So both of the choices multiply out to equal the original.

So $(3x-5)(2x+2)$ is not completely factored, because we can factor 2 out of $2x+2$.

$$(3x-5)(2x+2)$$
$$=(3x-5)(2)(x+1)$$
$$=\boxed{2(3x-5)(x+1)}$$ is the complete factorization

More examples from 6.3 (factoring trinomials)

Example: $2x^2-25x+12$

(+) same signs
want sum of $25x$ for middle term
Factor.

$$\boxed{(2x - 1)(x - 12)}$$

Check: $2x^2-24x-1x+12$

$2x^2-25x+12$ ✓

$2x^2$ 12

$2x\cdot x$ $1\cdot 12$, $2\cdot 6$, $3\cdot 4$

$1x$ $24x$

Ex: $4x^2+16x-9$

(−) signs are opposite
want a difference of $16x$ for middle term

$$\boxed{(2x - 1)(2x + 9)}$$

Check: $4x^2+18x-2x-9$

$=4x^2+16x-9$

$4x^2$ 9

$4x\cdot x$, $2x\cdot 2x$ $1\cdot 9$, $3\cdot 3$

$2x$ $18x$

<u>Try these:</u>

1) $6x^2 + 19x + 14$

2) $7x^2 - 10x + 3$

3) $10x^2 + 49x - 5$

4) $2x^2 - 11xy + 5y^2$

<u>Ex:</u> $18x^3 + 15x^2 - 18x$

$= 3x(6x^2 + 5x - 6)$

$= \boxed{3x(2x + 3)(3x - 2)}$

(−) signs are <u>opposite</u>
want a <u>difference</u>
of $5x$

$6x^2$: $6x \cdot x$, $2x \cdot 3x$

6: $1 \cdot 6$, $2 \cdot 3$

$9x$

$4x$

<u>Check:</u> $(2x+3)(3x-2)$

$= 6x^2 - 4x + 9x - 6$

$= 6x^2 + 5x - 6$ ✓ok

<u>Ex:</u> $-6x^2 + 11x - 4$

$= -1(6x^2 - 11x + 4)$

$= -1(2x - 1)(3x - 4)$

(+) <u>same</u> signs
want <u>sum</u> of $11x$

$6x^2$: $6x \cdot x$, $2x \cdot 3x$

4: $1 \cdot 4$, $2 \cdot 2$

$3x$

$8x$

<u>Check:</u> $(2x-1)(3x-4)$

$= 6x^2 - 8x - 3x + 4$

$= 6x^2 - 11x + 4$

Final answer: $\boxed{-(2x - 1)(3x - 4)}$

6.4: Factoring trinomials (leading coefficient >1) using the AC method:

Work the 6.4 HW the same way as the 6.3 homework.

6.5: Difference of Two Squares and Perfect Square Trinomials

Recall: $(a+b)(a-b)$
$$= a^2 - \cancel{ab} + \cancel{ab} - b^2$$
$$= a^2 - b^2$$

Difference of Two Squares Factorization

$$a^2 - b^2 = (a+b)(a-b)$$

Ex: Factor.

$$x^2 - 25$$
$$= x^2 - 5^2$$
$$= \boxed{(x+5)(x-5)}$$

Check: $x^2 - \cancel{5x} + \cancel{5x} - 25$
$$= x^2 - 25 \checkmark$$

Perfect Squares:

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$
$12^2 = 144$

Example: Factor.

$$4x^2 - 9$$
$$= (2x)^2 - (3)^2$$
$$= \boxed{(2x+3)(2x-3)} \text{ or } \boxed{(2x-3)(2x+3)}$$

Check:
$4x^2 - 6x + 6x - 9 = 4x^2 - 9 \checkmark$

Ex: $100y^2 - 81z^2$

$(10y)^2 - (9z)^2$

$\boxed{(10y + 9z)(10y - 9z)}$

Check it!

Ex: $32x^2 - 50y^2$

$= 2(16x^2 - 25y^2)$

$= 2((4x)^2 - (5y)^2)$

$= \boxed{2(4x - 5y)(4x + 5y)}$

Check: $(4x - 5y)(4x + 5y)$

$= 16x^2 + 20\cancel{xy} - 20\cancel{xy} - 25y^2$

$= 16x^2 - 25y^2$ ✓

Ex: $x^5 - x$

$x(x^4 - 1)$

$x((x^2)^2 - (1)^2)$

$x(x^2 - 1)(x^2 + 1)$

$x(x+1)(x-1)(x^2+1)$

$\boxed{x(x+1)(x-1)(x^2+1)}$

Note: $x^2 + 1$ is prime

Ex: $x^2 + 49$

Try: $(x+7)(x+7)$

$x^2 + 7x + 7x + 49$

$= x^2 + 14x + 49$ No!

Try: $(x+7)(x-7)$

$x^2 - 7\cancel{x} + 7\cancel{x} - 49$

$x^2 - 49$ No!

$\boxed{x^2 + 49 \text{ is Prime.}}$

Important Fact: $a^2 + b^2$ is always prime.

$x^2 +$ positive is always prime.

<u>Note</u>: $x^2 + 49$

(+) <u>same</u> signs

Rewrite: $x^2 + 0x + 49$

want <u>sum</u> of $0x$

x^2 49

$x \cdot x$ $7 \cdot 7$

$7x$

$7x$

sum is $14x$

It's impossible to get a sum of 0.