Additional examples from 6.3.

Note Title 3/22/2017

Ex: Factor.

$$x^4 + 8x^2 + 15$$

$$\boxed{(x^2 + 3)(x^2 + 5)}$$

x^4 $\quad$ 15

$x^2 \cdot x^2$ $\quad$ $1 \cdot 15$, $3 \cdot 5$

$3x^2$

$5x^2$

Check: $x^4 + 5x^2 + 3x^2 + 15$

$x^4 + 8x^2 + 15$ ✓

(+) same signs
want a sum of $19x^2$

Ex: $2x^4 - 19x^2 + 9$

$(2x^2 - 1)(x^2 - 9)$

$2x^4$ $\quad$ 9

$2x^2 \cdot x^2$ $\quad$ $1 \cdot 9$, $3 \cdot 3$

$1x^2$

$18x^2$

Check: $2x^4 - 18x^2 - 1x^2 + 9$

$= 2x^4 - 19x^2 + 9$ ✓

It factors more!

$(2x^2 - 1)(x^2 - 9)$

$$\boxed{(2x^2 - 1)(x+3)(x-3)}$$

Recall:

Difference of 2 Squares

$$a^2 - b^2 = (a+b)(a-b)$$

$x^2 - 9$

$x^2 - 3^2$

$(x+3)(x-3)$

Recall:

$x^2 +$ positive is always prime

6.5: Difference of 2 Squares & Perfect Square Trinomials

Note: $x^2 + 9$ is $\boxed{\text{prime.}}$

$x^2 + 0x + 9$ (+) same signs, sum of $0x$ (write as a trinomial)

x^2 ∧ $x \cdot x$

9 ∧ $1 \cdot 9$, $3 \cdot 3$

It's not possible to get a sum of 0.

Similarly: $2x^2 + 5$ is prime

$4x^2 + 25$ is prime

Ex: $4x^2 + 18$

$\boxed{2(2x^2 + 9)}$

Ex: $x^4 - 13x^2 + 36$

$(x^2 - 4)(x^2 - 9)$ Check it!

$\boxed{(x+2)(x-2)(x+3)(x-3)}$

x^4 ∧ $x^2 \cdot x^2$

36 ∧ $1 \cdot 36$, $2 \cdot 18$, $3 \cdot 12$, $4 \cdot 9$, $6 \cdot 6$

$9x^2$

$4x^2$

Perfect Square Trinomials (we're still in 6.5)

Ex: Factor.

$x^2+12x+36$

$(x+6)(x+6)$

$\boxed{(x+6)^2}$

Note: $x \cdot x$ is usually written x^2.

$(x+6)(x+6)$ is written $(x+6)^2$

Ex: Factor.

$16x^2+24x+9$

Try: $\boxed{(4x+3)^2}$

Check: $(4x+3)(4x+3)$

$16x^2+12x+12x+9$

$16x^2+24x+9$ ✓

$$\boxed{\begin{array}{l}\underline{\text{Perfect Squares:}}\\ (a+b)^2 = a^2+2ab+b^2\\ (a-b)^2 = a^2-2ab+b^2\end{array}}$$

Check:

$(a+b)(a+b)$

$=a^2+\underline{ab}+\underline{ab}+b^2$

$=a^2+2ab+b^2$

Ex: $16x^2-30x+9$

Try: $(4x-3)^2$

Check: $(4x-3)(4x-3)$

$16x^2-12x-12x+9$

$16x^2-24x+9$ No!

Start over: $16x^2-30x+9$

(+) same signs

sum of 30x

$\boxed{(8x-3)(2x-3)}$

$16x^2$	9
$\wedge$	$\wedge$
$16x \cdot x$	$1 \cdot 9$
$8x \cdot 2x$	$3 \cdot 3$
$4x \cdot 4x$	

6x

24x

Perfect Squares:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

We can use this pattern to square binomials in one step:

Ex: $(x+6)^2$

$$\boxed{x^2 + 12x + 36}$$

Ex: $(x-9)^2$

$$\boxed{x^2 - 18x + 81}$$

Check: $(x-9)(x-9)$
$= x^2 - 9x - 9x + 81$
$= x^2 - 18x + 81$

Ex: $(4a-7b)^2$

$$\boxed{16a^2 - 56ab + 49b^2}$$

6.6: Sum and Difference of Two Cubes

Sum and Difference of Two Cubes Factorization

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Mnemonic Device (for remembering the signs)

M Match
O Opposite
P Positive

S Same
O Opposite
A Always
P Positive

Perfect Cubes

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
$10^3 = 1000$

Example. Factor.

$$y^3 - 27$$
$$= y^3 - 3^3$$
$$= \boxed{(y-3)(y^2 + 3y + 9)}$$

Here, we use
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
with $a = y$, $b = 3$

Check: $y^3 + \cancel{3y^2} + \cancel{9y}$
$\quad -\cancel{3y^2} - \cancel{9y} - 27$
$= y^3 - 27$ ✓ok

Ex: Factor.

$$8x^3 + 125y^3$$

$$(2x)^3 + (5y)^3$$

$$= (2x + 5y)\left((2x)^2 - 2x(5y) + (5y)^2\right)$$

$$= \boxed{(2x+5y)(4x^2 - 10xy + 25y^2)}$$

Check: $8x^3 - \cancel{20x^2y} + \cancel{50xy^2} + \cancel{20x^2y} - \cancel{50xy^2} + 125y^3$

$$= 8x^3 + 125y^3$$

Common mistake:

$$(2x+5y)(2x^2 - 10xy + 5y^2)$$

forgot to square the coefficient as well as the variable.

Ex: $8x^4 - 27x$

$$x(8x^3 - 27)$$

$$= x\left((2x)^3 - (3)^3\right)$$

$$= x(2x-3)\left((2x)^2 + 2x(3) + 3^2\right)$$

$$= \boxed{x(2x-3)(4x^2+6x+9)}$$

Check: $(2x-3)(4x^2+6x+9)$

$$= 8x^3 + \cancel{12x^2} + \cancel{18x} - \cancel{12x^2} - \cancel{18x} - 27$$

$$= 8x^3 - 27 \checkmark$$

$$x(8x^3 - 27) = 8x^4 - 27x \checkmark$$

Ex: $2x^4 + 5x^3 - 2x - 5$

[Factor by grouping]

$$= (2x^4 + 5x^3) + (-2x - 5)$$

$$= x^3(2x+5) - 1(2x+5)$$

$$= (2x+5)(x^3 - 1)$$

[Notice: $x^3 - 1$ is a difference of 2 cubes]

$$x^3 - 1$$

$$= x^3 - 1^3$$

$$= (2x+5)(x-1)(x^2 + 1x + 1^2)$$

$$= \boxed{(2x+5)(x-1)(x^2+x+1)}$$