More examples:

Note Title 3/27/2017

Ex: $8z^3 + 125w^3$

Use $a^3+b^3 = (a+b)(a^2-ab+b^2)$

$$= (2z)^3 + (5w)^3$$

$$= (2z+5w)\left((2z)^2 - 2z(5w) + (5w)^2\right)$$

$$= \boxed{(2z+5w)(4z^2-10zw+25w^2)}$$

Check it!

Check: $8z^3 - \cancel{20z^2w} + \cancel{50zw^2} + \cancel{20wz^2} - \cancel{50zw^2} + 125w^3$

$$= 8z^3 + 125w^3 \checkmark \text{ok}$$

$a^2-b^2 = (a+b)(a-b)$

Ex: $a^6 - b^9$

$$= (a^2)^3 - (b^3)^3$$

$$= (a^2-b^3)\left((a^2)^2 + a^2b^3 + (b^3)^2\right)$$

$$= \boxed{(a^2-b^3)(a^4+a^2b^3+b^6)}$$

Ex: $81x^4 - y^4$

$$= (9x^2)^2 - (y^2)^2$$

$$= \underbrace{(9x^2+y^2)}_{a+b}\underbrace{(9x^2-y^2)}_{a-b}$$

$$= \boxed{(9x^2+y^2)(3x-y)(3x+y)}$$

Ex: $a^9 + b^9$

$$= (a^3)^3 + (b^3)^3$$

$$= (a^3+b^3)\left((a^3)^2 - a^3b^3 + (b^3)^2\right)$$

$$= \underbrace{(a^3+b^3)}(a^6-a^3b^3+b^6)$$

this factors more!

$$= \boxed{(a+b)(a^2-ab+b^2)(a^6-a^3b^3+b^6)}$$

Check: $(a^3+b^3)(a^6-a^3b^3+b^6)$

$$= a^9 - \cancel{a^6b^3} + \cancel{a^3b^6} + \cancel{b^3a^6} - \cancel{a^3b^6} + b^9 = a^9+b^9 \checkmark$$

Recall

How can a binomial factor:

* GCF
* Difference of 2 squares
* Sum of 2 cubes
* Difference of 2 cubes

Note: A sum of 2 squares is prime, unless it is also a sum of 2 cubes

6.7: Solving Quadratic Equations by Factoring

Definition:

A <u>quadratic equation (in x)</u> is an equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.

$$\underbrace{ax^2}_{\text{quadratic term}} + \underbrace{bx}_{\text{linear term}} + \underbrace{c}_{\text{constant term}} = 0$$

is <u>standard form</u> for a quadratic equation.

Zero Product Property of Numbers (Zero Product Theorem)

If the product of two numbers is 0, then at least one of the numbers must be 0.

In other words, if $AB = 0$, then $A = 0$ or $B = 0$.

Reminder:

Expressions get simplified (or factored) [Result is an expression, likely with variables]

Equations get <u>solved</u> [Result is a set of numbers, the solution set of the equation]

Ex:

Solve.

$$x^2 - 24 = 10x$$
$$\quad -10x \qquad -10x$$

$$x^2 - 10x - 24 = 0$$ (−) opp. signs difference of 10x

[Rewrite in standard form $ax^2+bx+c=0$]

$$\underbrace{(x-12)}_{A}\underbrace{(x+2)}_{B}=0$$

[Factor] x^2: $x \cdot x$; 24: $1\cdot 24$, $2\cdot 12$, $3\cdot 8$, $4\cdot 6$; $2x$, $12x$

$x - 12 = 0$ or $x + 2 = 0$

$+12 \quad +12$ | $-2 \quad -2$

$x = 12$ | $x = -2$

[Apply Zero Product Property by setting each Factor equal to 0]

Check: $(x-12)(x+2)$
$x^2 + 2x - 12x - 24$
$x^2 - 10x - 24$

Solution Set: $\boxed{\{12, -2\}}$

Check: Original eqn: $x^2 - 24 = 10x$

$x = 12$:
$(12)^2 - 24 = 10(12)$
$144 - 24 = 120$
$120 = 120$ ✓ True

$x = -2$:
$(-2)^2 - 24 = 10(-2)$
$4 - 24 = -20$
$-20 = -20$
✓ True

Steps for solving a quadratic equation

1) write equation in standard form $ax^2+bx+c=0$.
2) Factor the nonzero side.
3) Set each factor equal to 0. [Apply the Zero Product Property]
4) Solve the resulting linear equations.

Example: Solve.

$$6x^2 - 7x = 5$$

$(-)$ opp. signs
want a difference

$$6x^2 - 7x - 5 = 0$$

$$(2x + 1)(3x - 5) = 0$$

$6x^2$ $\quad$ 5

$6x \cdot x$ $\quad$ $1 \cdot 5$

$2x \cdot 3x$

$2x + 1 = 0$ OR $3x - 5 = 0$

$2x = -1$ $\qquad$ $3x = 5$

$\frac{2x}{2} = \frac{-1}{2}$ $\qquad$ $\frac{3x}{3} = \frac{5}{3}$

$x = -\frac{1}{2}$ $\qquad$ $x = \frac{5}{3}$

Check it!

$6x^2 - 10x + 3x - 5$

$6x^2 - 7x - 5$ ✓

Sol'n Set: $\boxed{\left\{-\frac{1}{2}, \frac{5}{3}\right\}}$

Example: Solve.

$$2x^3 - 14x^2 + 24x = 0$$

[This is not a quadratic eqn. This is a cubic (3rd degree eqn.)]

$$2x(x^2 - 7x + 12) = 0$$

$$2x(x - 3)(x - 4) = 0$$

[Set each factor equal to 0]

$2x = 0$ OR $x - 3 = 0$ OR $x - 4 = 0$

$\quad +3 \quad +3 \qquad +4 \quad +4$

$\frac{2x}{2} = \frac{0}{2}$ $\qquad$ $x = 3$ $\qquad$ $x = 4$

$x = 0$

Sol'n Set: $\boxed{\{0, 3, 4\}}$

Ex: Solve.

$$4x + 70 = 2x^2$$
$$\quad -2x^2 \qquad -2x^2$$

$$-2x^2 + 4x + 70 = 0$$
$$-2(x^2 - 2x - 35) = 0$$
$$-2(x + 5)(x - 7) = 0$$

$-2 = 0$ OR $x + 5 = 0$ OR $x - 7 = 0$

Never true! | $x = -5$ | $x = 7$

Solution Set: $\boxed{\{-5, 7\}}$