6.8: Applications of Quadratic Eqns (cont'd)

Note Title 4/3/2017

Example: If five is added to three times the square of a number, the result is sixteen times the number. Find all such numbers.

The unknown number: x

$$3x^2 + 5 = 16x$$

$-16x \quad -16x$

(+) same signs, sum of $-16x$

$3x^2 \qquad 5$

$3x \cdot x \qquad 1 \cdot 5$

$1x \qquad 15x$

$$3x^2 - 16x + 5 = 0$$

$$(3x - 1)(x - 5) = 0$$

Check:

$3x^2 - 15x - x + 5$

$= 3x^2 - 16x + 5$ ✓

$3x - 1 = 0$ OR $x - 5 = 0$

$+1 \quad +1 \qquad +5 \quad +5$

$3x = 1 \qquad x = 5$

$\frac{3x}{3} = \frac{1}{3}$

$x = \frac{1}{3}$

The numbers are 5 and $\frac{1}{3}$.

Example: The sum of the squares of two consecutive odd integers $\overset{=}{\text{is}}$ nineteen more than the product of the integers. Find the integers.

1st integer: x

2nd integer: $x+2$

$$\underbrace{x^2 + (x+2)^2}_{\text{sum of their squares}} = \underbrace{x(x+2)}_{\text{product}} + 19$$

$$x^2 + (x+2)(x+2) = x^2 + 2x + 19$$

$$x^2 + x^2 + 2x + 2x + 4 = x^2 + 2x + 19$$

$$\begin{aligned} 2x^2 + 4x + 4 &= x^2 + 2x + 19 \\ -x^2 \quad -2x \quad -19 & \quad -x^2 \quad -2x \quad -19 \end{aligned}$$

$$x^2 + 2x - 15 = 0$$

$$(x - 3)(x + 5) = 0$$

$x - 3 = 0$ or $x + 5 = 0$

$x = 3$ | $x = -5$

$x = 3$

1st integer: $x = 3$

2nd integer: $x+2$

substitute $x = 3 \Rightarrow 3+2 = 5$

The integer pair is 3, 5.

$x = -5$

1st integer: $x = -5$

2nd integer: $x+2$

substitute $x = -5 \Rightarrow -5+2 = -3$

The integer pair is $-5, -3$.

$\boxed{\text{The pair of consecutive integers is either } 3, 5 \text{ or } -5, -3.}$

Example: The length of a rectangular garden is 2 ft more than the width. The area is 120 ft^2. What are the length and width?

length $\xrightarrow[\text{to}]{\text{compare}}$ width

x

width: x

length: $x+2$

(length)(width) = Area

$(x+2)(x) = 120$

$x(x+2) = 120$

$x^2 + 2x = 120$

$x^2 + 2x - 120 = 0$

$(x+12)(x-10) = 0$

$x+12=0$ OR $x-10=0$

$x=-12$ $\quad$ $x=10$

Negative number does not make sense for width

Discard $x=-12$

$\underline{x=10}$ width: $x=10$

length: $x+2$

substitute $x=10 \Rightarrow 10+2=12$

$\boxed{\text{The width is 10 ft and the length is 12 ft.}}$

Example: The height of a triangle is 3 feet less than $\underbrace{\text{five times its base}}_{5x}$. The area of the triangle is 13 ft^2. Find the base and height.

$$\text{height} \xrightarrow[\text{to}]{\text{compare}} \text{base}$$
x

base: x

height: $5x-3$

$$\text{Area of triangle} = \frac{1}{2}(\text{base})(\text{height})$$

$$13 = \frac{1}{2}(x)(5x-3)$$

$$\frac{1}{2}x(5x-3) = 13$$

multiply both sides by 2 to clear the fraction:

$$\frac{\cancel{2}}{1}\left(\frac{1}{\cancel{2}}x\right)(5x-3) = 13(2)$$

$$x(5x-3) = 26$$

$$5x^2 - 3x = 26$$

$$5x^2 - 3x - 26 = 0$$

$5x^2$ 26

$5x \cdot x$ $1 \cdot 26$

$2 \cdot 13$

$13x$

$10x$

Check:

$5x^2 + 10x - 13x - 26$

$5x^2 - 3x - 26$ ✓

$$(5x - 13)(x + 2) = 0$$

$5x - 13 = 0$ OR $x + 2 = 0$

$5x = 13$

$\frac{5x}{5} = \frac{13}{5}$

$x = -2$

discard $x = -2$ (negative won't work for a dimension)

$$x = \frac{13}{5} = 2\frac{3}{5}$$

base: $x = \frac{13}{5}$

height: $5x-3$

substitute $x = \frac{13}{5} \Rightarrow \frac{\cancel{5}}{1}\left(\frac{13}{\cancel{5}}\right) - 3 = 13 - 3 = 10$

See next page

The base is $2\frac{3}{5}$ ft and the height is 10 ft.

Check (for previous problem): base: $2\frac{3}{5} = \frac{13}{5}$

5 times base: $5\left(\frac{13}{5}\right) = 13$

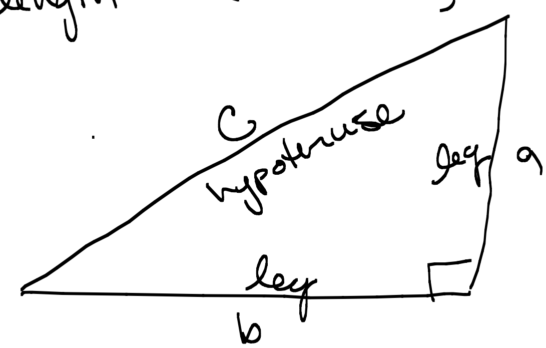
3 ft less: 10 ft ✓ (same as height)

Area: $\frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}\left(\frac{13}{5}\text{ ft}\right)(10\text{ ft}) = \frac{130}{10}\text{ ft}^2 = 13\text{ ft}^2$ ✓ ok

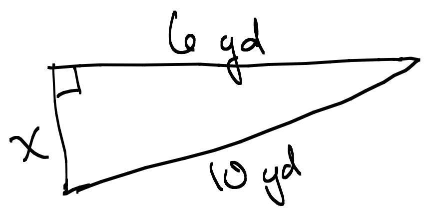
The Pythagorean Theorem:

In a right triangle with hypotenuse of length c and legs of length a and b,

then $a^2 + b^2 = c^2$.



Ex: Find the length of the missing side.



Here, $c = 10$

$a^2 + b^2 = c^2$

$x^2 + 6^2 = 10^2$

$x^2 + 36 = 100$

$\quad\;\; -100 \quad -100$

$x^2 + 36 - 100 = 0$

$x^2 - 64 = 0$

$(x+8)(x-8) = 0$

or

$x + 8 = 0$	$x - 8 = 0$
$x = -8$	$x = 8$
Discard it (negative dimension)	

The missing side is 8 yards.

7.1: Simplifying Rational Expressions

Recall: Rational number: can be written as the ratio (quotient, fraction) of two integers.

Examples of rational numbers:

$0, \frac{5}{2}, 3, \frac{2}{3}, \frac{14}{3}, 2\frac{3}{5}, 0.35, -\frac{1}{4}, 0.\bar{3}$

$0 = \frac{0}{6}$, $3 = \frac{3}{1}$, $2\frac{3}{5} = \frac{13}{5}$, $0.35 = \frac{35}{100}$, $0.\bar{3} = \frac{1}{3}$

(any decimal that ends or repeats can be written as a ratio)

Examples of Irrational Numbers

$\pi, \sqrt{2}, \sqrt{3}, e$

Rational Expression: can be written as the ratio (fraction) of 2 polynomials.

Examples of rational expressions:

$$\frac{x^2+4}{x^3-1}, \quad \frac{x^2+5x+3}{7-x^4}, \quad \frac{1}{x^2+3x}$$

To simplify a rational expression, we "cancel out" factors that appear in both the numerator and denominator.

Ex: $$\frac{8}{20} = \frac{4\cdot 2}{4\cdot 5} = \frac{\cancel{4}^{1}}{\cancel{4}}\cdot\frac{2}{5} = 1\cdot\frac{2}{5} = \boxed{\frac{2}{5}}$$

Ex:

$$\frac{\cancel{24}^{\,3}\, x^4 y^3}{\cancel{32}_{\,4}\, x^5 y} = \boxed{\frac{3y^2}{4x}}$$

Ex:

$$\frac{x^2+8x+15}{x^2+5x+6} = \frac{\cancel{(x+3)}^{\,1}(x+5)}{(x+2)\cancel{(x+3)}_{\,1}}$$

$$= \boxed{\frac{x+5}{x+2}}$$