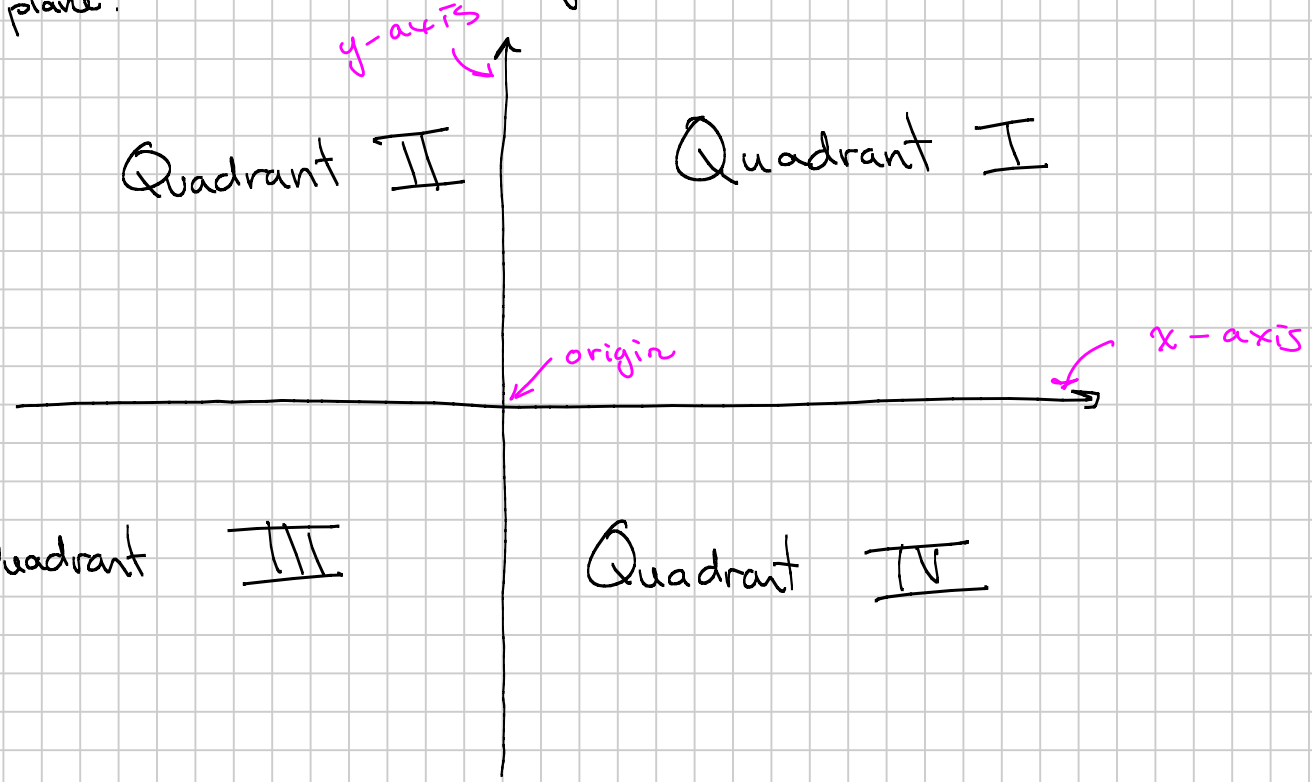
3.1: The Rectangular Coordinate System.

Note Title 2/5/2015

The rectangular coordinate system is also called the Cartesian coordinate system, the Cartesian plane, the xy-plane.



Example: Plot these points:

A point on the graph is represented by an ordered pair (x, y).

The first coordinate is the x-coordinate.

The second coordinate is the y-coordinate.

$(2, 4)$

$(-3, 1)$

$(-4, -5)$

$(3, -2)$

$(0, 0)$

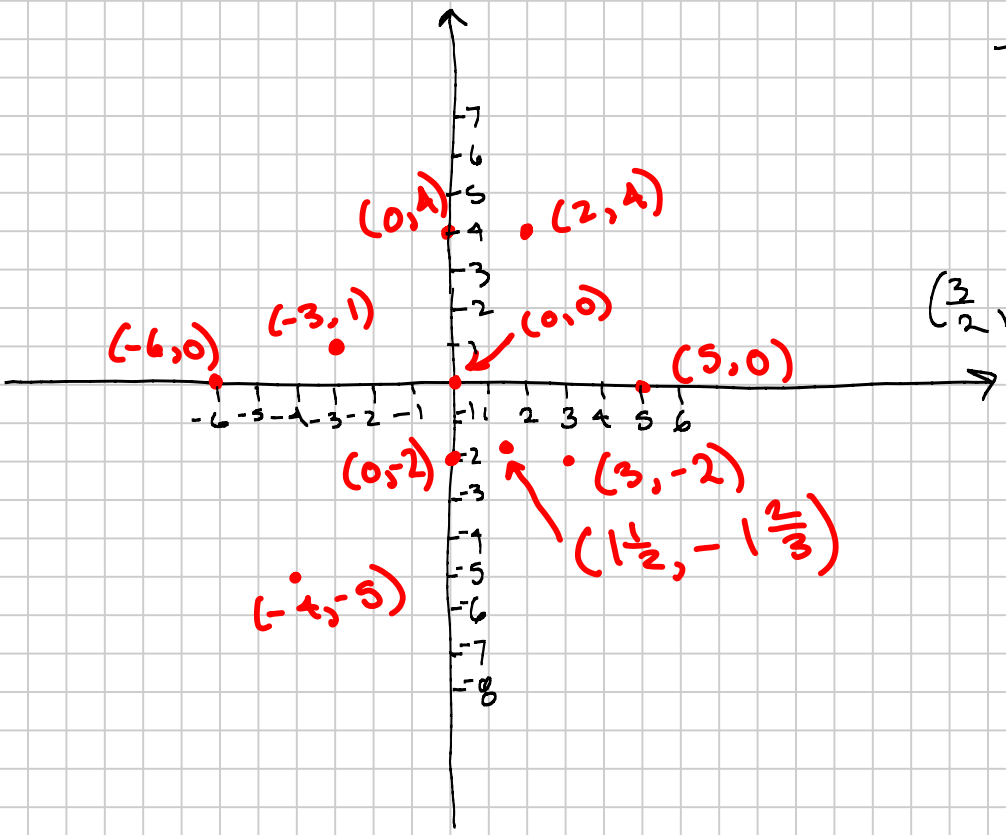
$(0, 4)$

$(5, 0)$

$(-6, 0)$

$(0, -2)$

$\left(\frac{3}{2}, -\frac{5}{3}\right)$



To graph $\left(\frac{3}{2}, -\frac{5}{3}\right)$, convert to mixed numbers:

$\left(\frac{3}{2}, -\frac{5}{3}\right) = \left(1\frac{1}{2}, -1\frac{2}{3}\right)$

3.2: Linear Equations in Two Variables

A **linear equation in two variables** can be written as $Ax + By = C$, where A, B, and C are real numbers.

A **solution** to a linear equation in two variables is an ordered pair that makes the equation **True**.
(It gives a true statement when the values in the ordered pair are substituted into the equation)

Example: Is $(4, 6)$ a solution to $3x - 2y = 1$?

Substitute $x = 4$, $y = 6$: $3(4) - 2(6) = 1$

$12 - 12 = 1$

$0 = 1$ False

No, $(4, 6)$ is not a solution.

Ex: Is $(5, 8)$ a solution to $3x - 2y = 1$?

Substitute $x = 5$, $y = 8$: $3(5) - 2(8) = 1$

$15 - 16 = 1$

$-1 = 1$ False

No, $(5, 8)$ is not a solution.

Ex: Is $(5, 7)$ a solution to $3x - 2y = 1$?

$x = 5$, $y = 7 \Rightarrow$ $3(5) - 2(7) = 1$

$15 - 14 = 1$

$1 = 1$ True ✓

Yes, $(5, 7)$ is a solution.

Ex: Is $(3, 4)$ a solution to $3x - 2y = 1$?

$x = 3$, $y = 4 \Rightarrow$ $3(3) - 2(4) = 1$

$9 - 8 = 1$

$1 = 1$ ✓ True!

Yes, $(3, 4)$ is a solution.

There are infinitely many solutions to this equation.

Same example: $3x - 2y = 1$

Try $(1,1)$: $x=1,\ y=1 \Rightarrow$ $3(1) - 2(1) = 1$

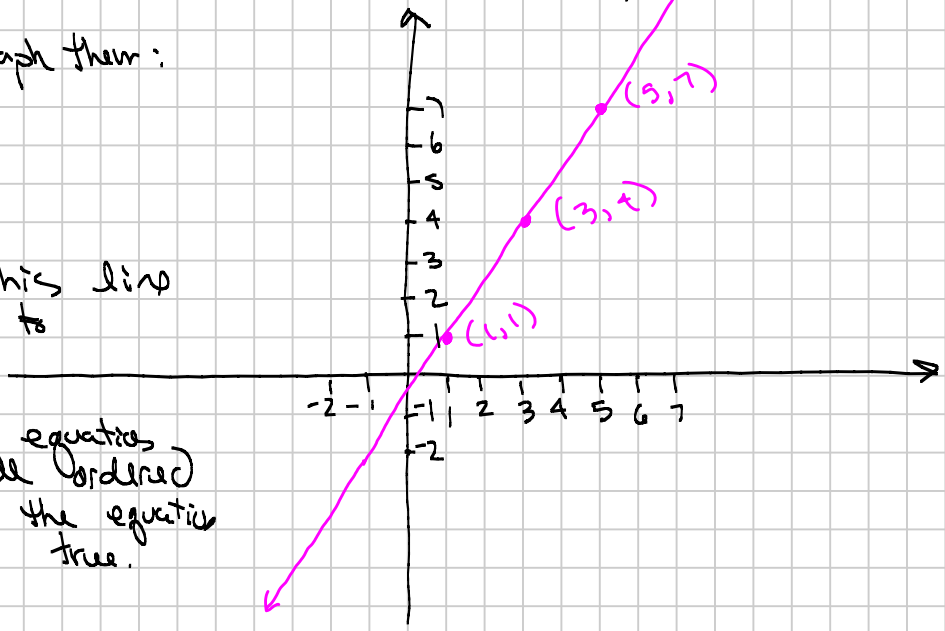
$3 - 2 = 1$

$1 = 1$ True

Yes, $(1,1)$ is a solution.

So far, we have 3 solutions: $(5,7)$
$(3,4)$
$(1,1)$

Let's graph them:



Every point on this line is a solution to $3x - 2y = 1$

The graph of an equation is the set of all ordered pairs that make the equation true.

Example: Graph the line $-3x + 2y = -4$.

Choose a value of x or y, and then solve for the other.

$x = 2$ | $-3x + 2y = -4$

$x=2 \Rightarrow -3(2) + 2y = -4$

$-6 + 2y = -4$

$+6 \quad +6$

$2y = 2$

$\frac{2y}{2} = \frac{2}{2}$

$y = 1$

$(2,1)$

$x = 4$ | $-3x + 2y = -4$

$-3(4) + 2y = -4$

$-12 + 2y = -4$

$+12 \quad +12$

$2y = 8$

$\frac{2y}{2} = \frac{8}{2}$

$y = 4$ $(4,4)$

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Previous example cont'd. $-3x + 2y = -4$

$y = 9$

$-3x + 2(9) = -4$

$-3x + 18 = -4$

$-18 \quad -18$

$-3x = -22$

$\frac{-3x}{-3} = \frac{-22}{-3}$

$x = \frac{22}{3} = 7\frac{1}{3}$

$(7\frac{1}{3}, 9)$

$y = -5$

$-3x + 2(-5) = -4$

$-3x - 10 = -4$

$-3x = -4 + 10$

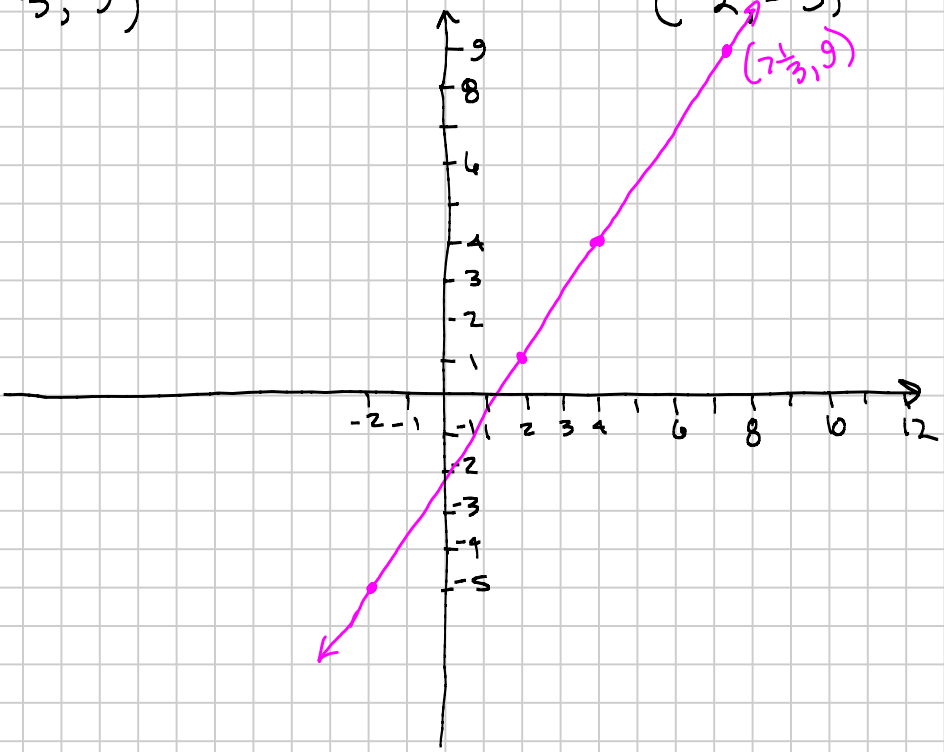
$-3x = 6$

$\frac{-3x}{-3} = \frac{6}{-3}$

$x = -2$

$(-2, -5)$

4 solutions: $(4,4)$, $(2,1)$, $(7\frac{1}{3}, 9)$, $(-2,-5)$



Sometime you'll see a table:

x	y
2	1
4	4
$7\frac{2}{3}$	9
-2	-5

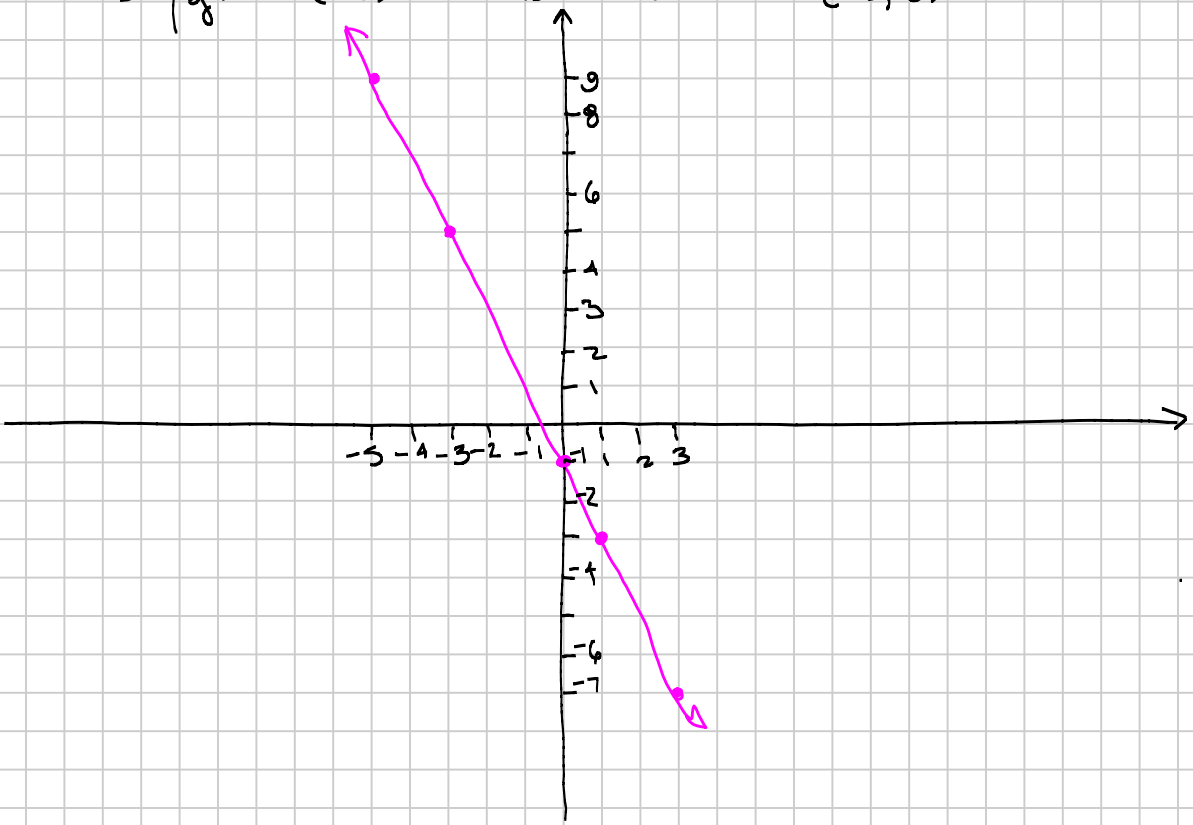
Example: Graph $2x+y=-1$ by solving for y first.

Isolate y: $2x+y=-1$

$-2x \quad -2x$

$y=-2x-1$

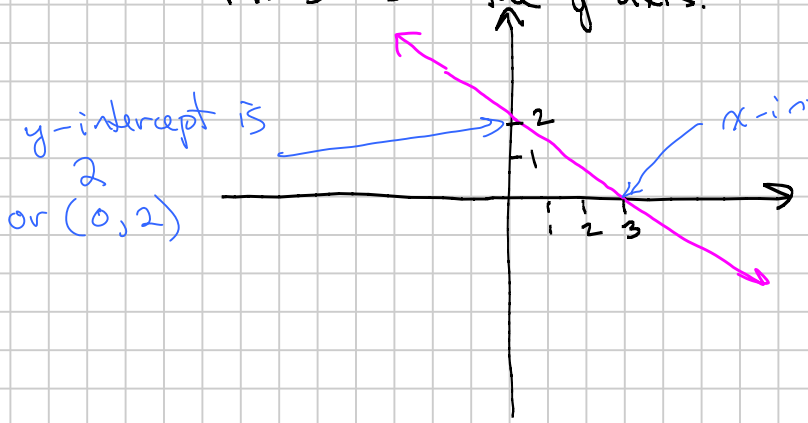
x	$y=-2x-1$	Pairs
0	$y=-2(0)-1=0-1=-1$	$(0,-1)$
1	$y=-2(1)-1=-2-1=-3$	$(1,-3)$
3	$y=-2(3)-1=-6-1=-7$	$(3,-7)$
-3	$y=-2(-3)-1=6-1=5$	$(-3,5)$
-5	$y=-2(-5)-1=10-1=9$	$(-5,9)$

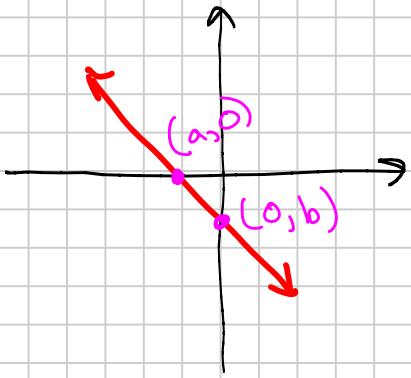


x-intercepts and y-intercepts

x-intercept: x-coordinate of the point where the graph intersects the x-axis.

y-intercept: y-coordinate of the point where the graph intersects the y-axis.





To find the x-intercept, set $y=0$ and solve for x
To find the y-intercept, set $x=0$ and solve for y.

Example: graph the line using intercepts.

$$-2x + 4y = 12$$

Find x-intercept:

Set $y=0$

$$\begin{aligned} -2x+4(0) &= 12 \\ -2x+0 &= 12 \\ -2x &= 12 \\ \frac{-2x}{-2} &= \frac{12}{-2} \\ x &= -6 \end{aligned}$$

x-intercept: -6 or $(-6,0)$

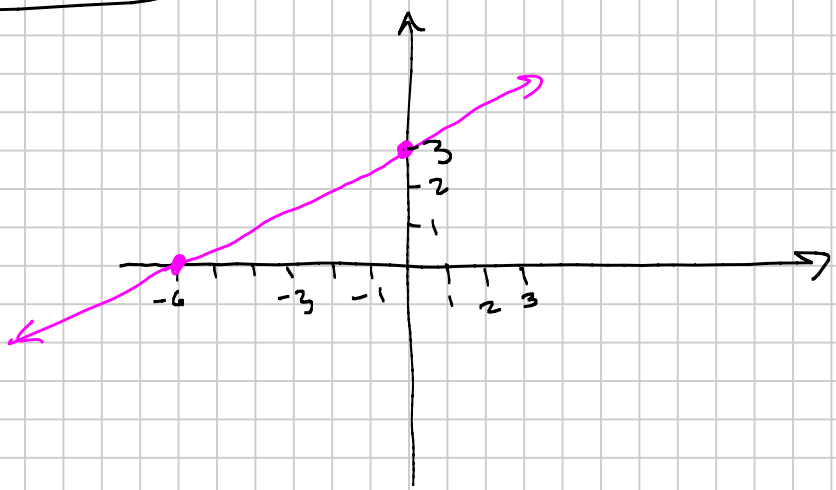
ordered pair: $(-6,0)$

Find y-intercept:

Set $x=0$

$$\begin{aligned} -2(0)+4y &= 12 \\ 0+4y &= 12 \\ 4y &= 12 \\ \frac{4y}{4} &= \frac{12}{4} \\ y &= 3 \end{aligned}$$

y-intercept: 3 or $(0,3)$



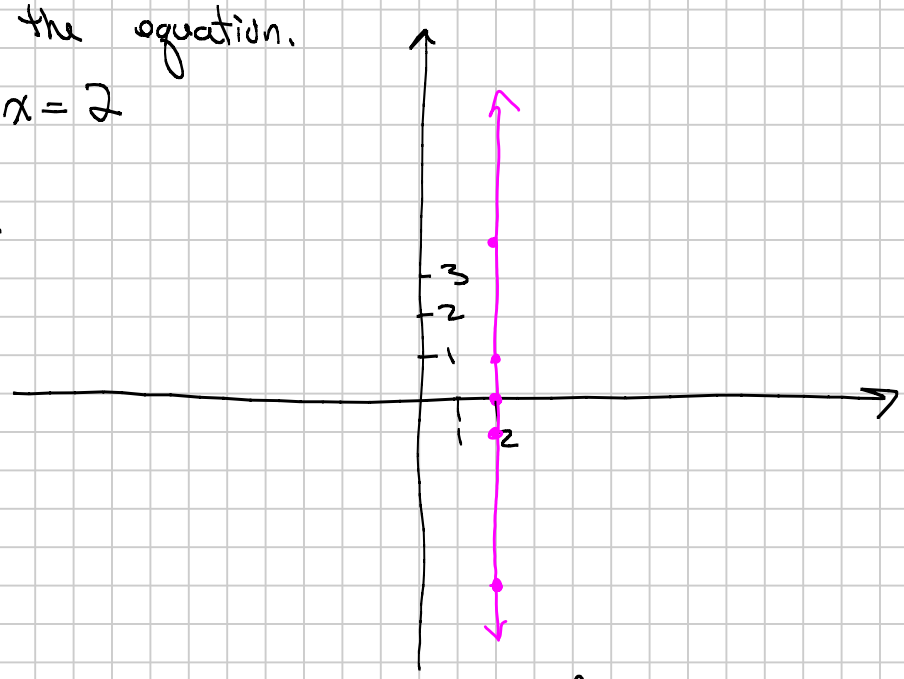
Horizontal and vertical lines

Ex: Graph the equation.

$x = 2$

Ordered pair solution

(2, 0)
(2, 1)
(2, 4)
(2, -5)
(2, -1)



Graph the eqn $y = -3$

ordered pairs

(1, -3)
(3, -3)
(2, -3)
(-4, -3)