3.3: Slope of a line (continued)

Note Title 2/5/2015

Definition

Slope $= m = \dfrac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are points on the line.

Example: Calculate the slope of the line passing through the points $(2, -4)$ and $(5, 7)$.

$(2, -4) = (x_1, y_1)$

$(5, 7) = (x_2, y_2)$

$x_1 = 2$, $y_1 = -4$

$x_2 = 5$, $y_2 = 7$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-4)}{5 - 2} = \frac{7 + 4}{3} = \frac{11}{3}$$

Slope $= \frac{11}{3}$

Does it matter which point I start with? No:

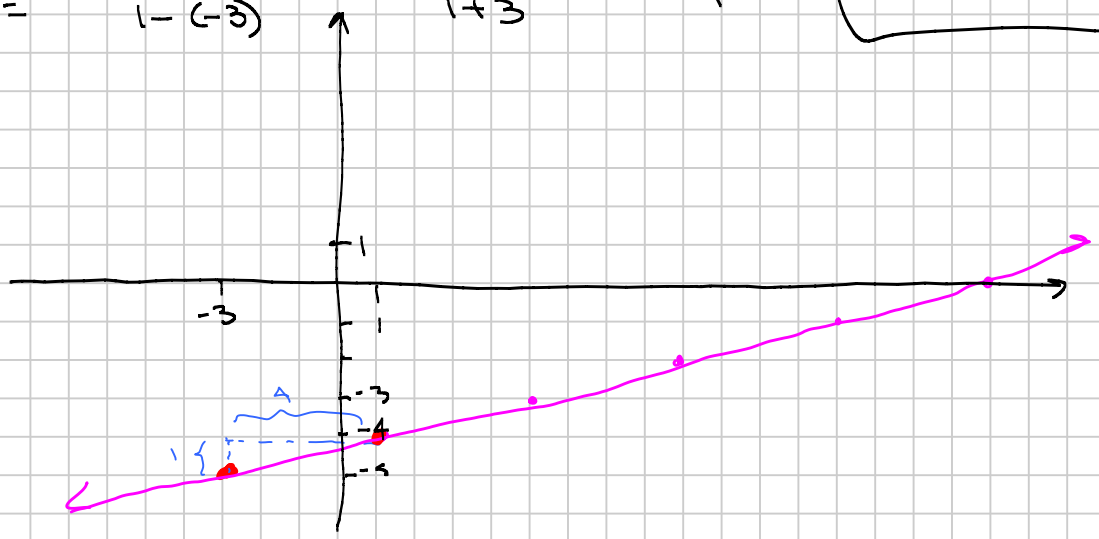
$$m = \frac{-4 - 7}{2 - 5} = \frac{-11}{-3} = \frac{11}{3}$$

equal

Example: Calculate the slope of the line passing through $(-3, -5)$ and $(1, -4)$. Graph it.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-5)}{1 - (-3)} = \frac{-4 + 5}{1 + 3} = \frac{1}{4}$$

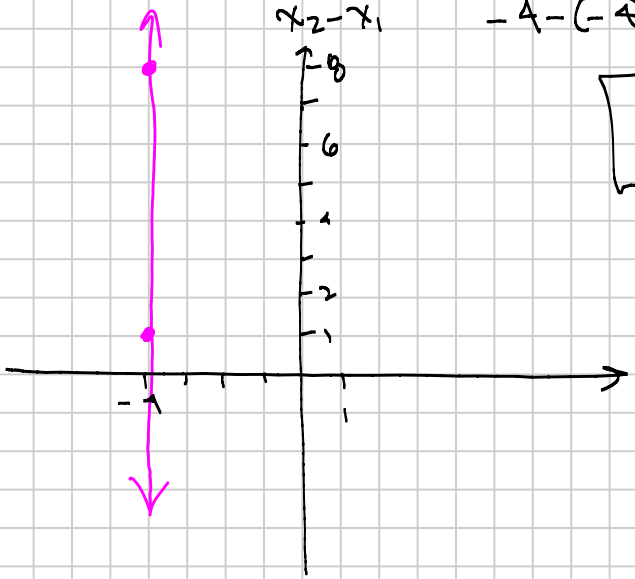
Slope $= \frac{1}{4}$



Ex: Find slope of line containing $(-4,1)$ and $(-4,8)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-1}{-4-(-4)} = \frac{7}{-4+4} = \frac{7}{0}$$ undefined!

Slope is undefined

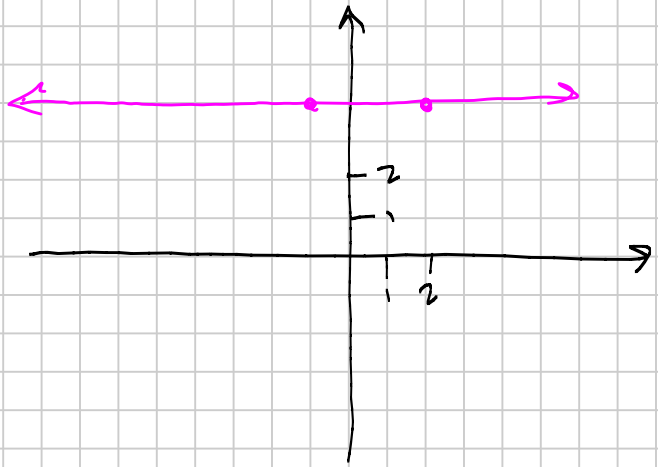


This is a vertical line.

Ex: Find the slope of the line containing $(2,3)$ and $(-1,3)$.

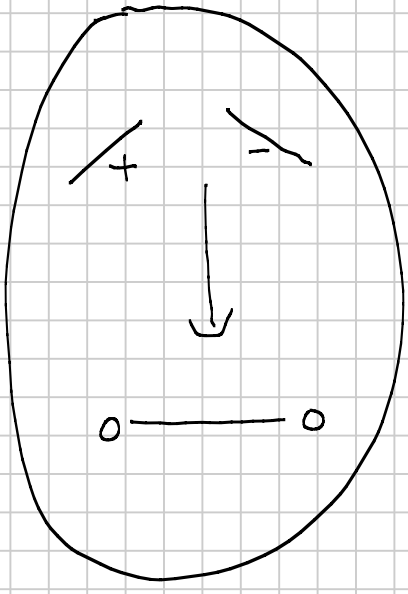
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-3}{-1-2} = \frac{0}{-3} = 0$$

Slope = 0



This is a horizontal line.

Mr.
Slope
Guy



Important:

* Every vertical line has undefined slope
* Every horizontal line has slope 0.

Parallel and Perpendicular lines:

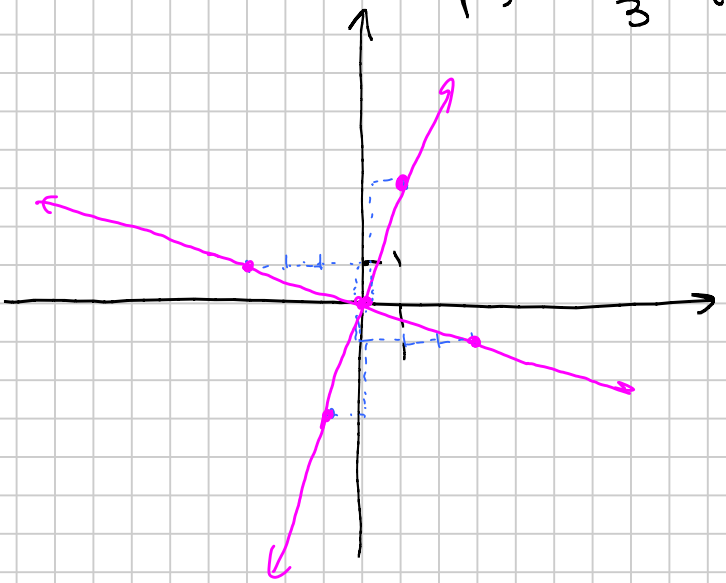
Parallel lines: never intersect

Perpendicular lines: intersect at a right angle ($90°$, square corner)

For lines that aren't vertical or horizontal:

* **Parallel lines** have the **same slope**
* **Perpendicular lines** have slopes that are opposite reciprocals of each other.

Why opposite reciprocals? Graph lines through origin with slopes $-\frac{1}{3}$ and $\frac{3}{1}$.



Ex: If a line has slope -5, what is the slope of a line (a) parallel (b) perpendicular to it?

a) Parallel line would have slope -5

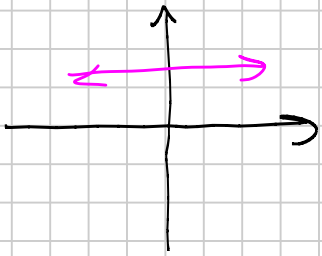
b) Perpendicular line would have slope $\frac{1}{5}$.

Given slope is $m_1 = -5 = -\frac{5}{1}$

Ex: If a line has slope 0, what is the slope of a line (a) parallel (b) perpendicular to it?

(a) Parallel line would have slope 0 also.

(b) Given line has slope 0, so it must be horizontal.



Perpendicular line must be vertical, so its slope is **undefined.**

3.4: Slope-intercept form of a linear equation

Standard form of a Line: $Ax + By = C$ where A and B are not both 0.

Slope-intercept form of a line: $y = mx + b$, where

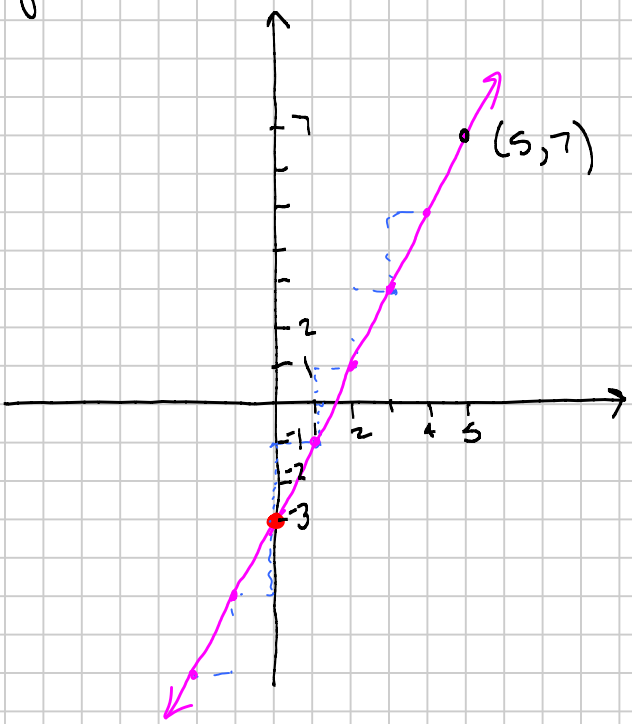
m is the slope and b is the y-intercept.

(or, (0, b) is the y-intercept)

Example: Graph the line $y = 2x - 3$.

Slope: $m = 2 = +\frac{2}{1}$

y-intercept: -3 or $(0, -3)$



Check: Is (5, 7) a solution?

$y = 2x - 3$

$x = 5, y = 7 \Rightarrow 7 = 2(5) - 3$

$7 = 10 - 3$

$7 = 7$ ✓ True! Yes.

Ex: Graph $5x = 2y$ by writing it in slope-intercept form.

$2y = 5x$

$\frac{2y}{2} = \frac{5x}{2}$

$y = \frac{5}{2}x$

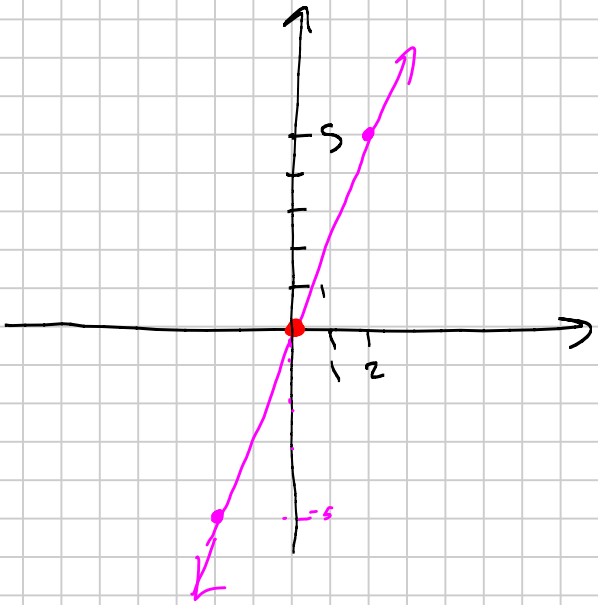
$y = \frac{5}{2}x + 0$ is $y = mx + b$ form

Slope: $m = \frac{5}{2}$; y-intercept: 0 or (0, 0)

See next page

Previous example cont'd:

$m = +\frac{5}{2}$

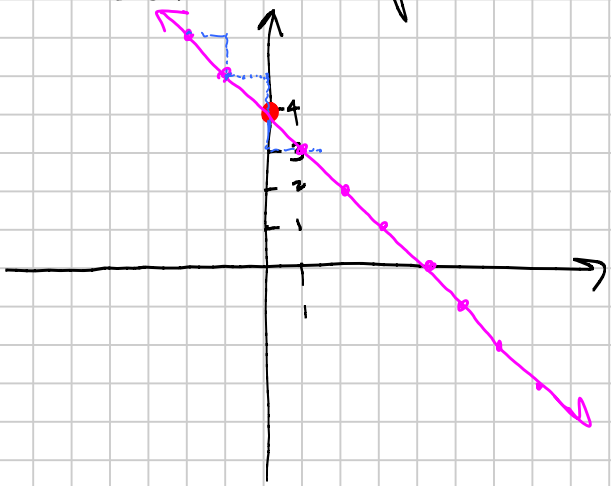


Ex: Graph $y = -x + 4$

Slope: $m = -1 = -\frac{1}{1}$

y-intercept: 4 or (0, 4)

can write $y = -1x + 4$



Example: write the equation in slope-intercept form.

$-3x + y = -4$.

$+3x \quad +3x$

$\boxed{y = 3x - 4}$

$m = 3$

Slope: $m = +\frac{3}{1}$

y-intercept: -4 or $(0, -4)$

Ex: write in slope-intercept form.

$4x - 7y = 12$

$-7y = -4x + 12$

$\frac{-7y}{-7} = \frac{-4x}{-7} + \frac{12}{-7}$

$\boxed{y = \frac{4}{7}x - \frac{12}{7}}$

slope: $m = \frac{4}{7}$, y-intercept: $(0, -\frac{12}{7})$

3.5: The Point-Slope Form of a Line

Point-slope form of a line: $y - y_1 = m(x - x_1)$,

where m is the slope and (x_1, y_1) is a point on the line.

Note: solving for m gives us:

$$\frac{y - y_1}{x - x_1} = \frac{m(x - x_1)}{x - x_1}$$

$$\frac{y - y_1}{x - x_1} = m$$

Example: Use the point-slope form to write the equation of the line with slope 5 and passing through the point $(-4, 3)$. Write answer in slope-intercept form.

$m = 5$, $x_1 = -4$, $y_1 = 3$

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 5(x - (-4))$$
$$y - 3 = 5(x + 4)$$
$$y - 3 = 5x + 20$$
$$+3 \qquad +3$$
$$\boxed{y = 5x + 23}$$

Ex: Write the equation of the line passing through the points $(-2, 3)$ and $(-6, 15)$.

Calculate the slope 1st:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15 - 3}{-6 - (-2)} = \frac{12}{-6 + 2} = \frac{12}{-4} = -3$$

Can use either of the given points for (x_1, y_1)

Using $(-2, 3)$: $x_1 = -2$, $y_1 = 3$

See next page

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -3(x - (-2))$$

$$y - 3 = -3(x + 2)$$

$$y - 3 = -3x - 6$$

$+3 \qquad +3$

$$\boxed{y = -3x - 3}$$