4.1: Solving linear systems by graphing (cont'd)

To solve a system by graphing, we graph both lines and use the graph to estimate the solution.

<u>Ex:</u> Solve the system by graphing.

$$x + 2y = 8$$
$$x - 3y = 3$$

<u>1st graph $x+2y=8$:</u>

Find x-intercept: set $y=0$: $x+2(0)=8$, $x+0=8$, $x=8$ — x-intercept: $(8,0)$

Find y-intercept: set $x=0$: $0+2y=8$, $2y=8$, $y=\frac{8}{2}=4$ — y-intercept $(0,4)$

OR write as $y=mx+b$:

$$x+2y=8$$
$$2y=-x+8$$
$$\frac{2y}{2}=\frac{-x}{2}+\frac{8}{2}$$
$$y=-\frac{1}{2}x+4$$

slope: $m=-\frac{1}{2}$

b: 4

<u>Next graph $x-3y=3$:</u>

Find x-intercept: set $y=0$: $x-3(0)=3$, $x-0=3$, $x=3$ — x-intercept: $(3,0)$

Find y-intercept: set $x=0$: $0-3y=3$, $-3y=3$, $\frac{-3y}{-3}=\frac{3}{-3} \Rightarrow y=-1$ — y-intercept: $(0,-1)$

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or, write as $y=mx+b$:

$$x-3y=3$$
$$-3y=-x+3$$
$$\frac{-3y}{-3}=\frac{-x}{-3}+\frac{3}{-3}$$
$$y=\frac{1}{3}x-1$$

Slope: $m=\frac{1}{3}$

y-int: $y=-1$

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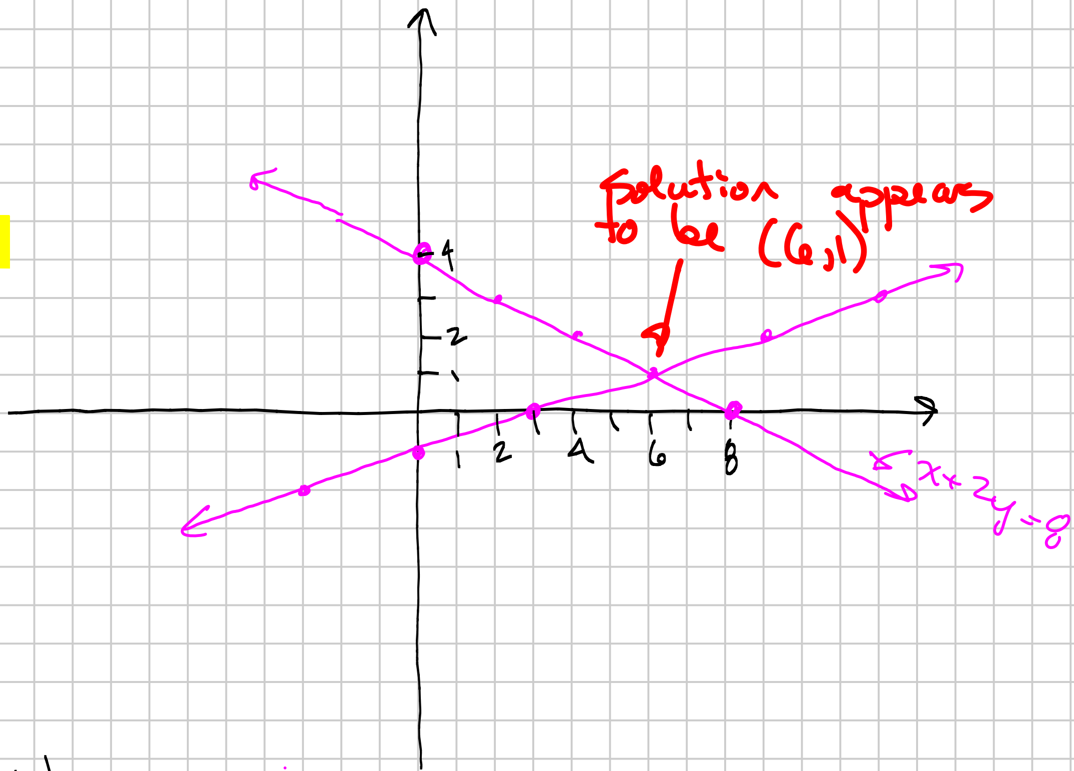
$x + 2y = 8$

$m = -\frac{1}{2}$

y int: $(0, 4)$

x-int: $(8, 0)$



$x - 3y = 3$

x-intercept: $(3, 0)$

y-intercept: $(0, -1)$

write as $y = mx + b$

$$x - 3y = 3$$
$$-x \qquad -x$$
$$-3y = -x + 3$$
$$\frac{-3y}{-3} = \frac{-x}{-3} + \frac{3}{-3}$$
$$y = \frac{1}{3}x - 1$$

slope: $m = +\frac{1}{3}$

y-intercept: $b = -1 \Rightarrow (0, -1)$

* when solving a system by graphing graph both lines and then obtain the solution from the graph.

For this example, Solution is $\boxed{(6, 1)}$.

Check it:

$x = 6,\ y = 1 \Rightarrow$

$$x + 2y = 8$$
$$6 + 2(1) = 8$$
$$6 + 2 = 8$$
$$8 = 8 \checkmark$$

$$x - 3y = 3$$

$x = 6,\ y = 1 \Rightarrow 6 - 3(1) = 3$

$$6 - 3 = 3$$
$$3 = 3 \checkmark$$

4.2: The substitution method (for solving linear systems)

Steps for solving by substitution

1) Solve one equation (your choice) for one variable (your choice)
2) substitute this result into the other equation.
3) Solve for the remaining variable.
4) Substitute this value into either equation and solve.
5) Check your solution! (by substituting your ordered pair into both eqns)

Example: Solve the system by substitution.

$$\begin{cases} x - y = 9 \\ 2x - 11y = -18 \end{cases}$$

1) Solve $x - y = 9$ for x:

$$x - y = 9$$
$$\quad +y \quad +y$$
$$x = 9 + y$$

2) substitute $x = 9 + y$ into $2x - 11y = -18$:

$$2(9+y) - 11y = -18$$

3) Solve for y:

$$18 + 2y - 11y = -18$$
$$18 - 9y = -18$$

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Previous example cont'd.:

$$18 - 9y = -18$$
$$-18 \qquad -18$$

$$-9y = -36$$

$$\frac{-9y}{-9} = \frac{-36}{-9}$$

$$y = 4$$

4) Put $y = 4$ into $x - y = 9$

$$x - 4 = 9$$
$$+4 \qquad +4$$

$$x = 13$$

Sol'n: $\boxed{(13, 4)}$

5) Check it: $x - y = 9$

$x = 13, y = 4 \Rightarrow 13 - 4 = 9$

$9 = 9$ ✓

$2x - 11y = -18$

$x = 13, y = 4$

$\Rightarrow 2(13) - 11(4) = -18$

$26 - 44 = -18$

$-18 = -18$ ✓

Example: Solve by substitution:

$$-4x + 3y = 19$$
$$-6x - 5y = 0$$

1) Solve $-6x - 5y = 0$ for y:

$$-5y = 6x$$

$$\frac{-5y}{-5} = \frac{6x}{-5}$$

$$y = -\frac{6}{5}x$$

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2) Substitute $y = -\frac{6}{5}x$ into $-4x + 3y = 19$

$$-4x + \frac{3}{1}\left(-\frac{6}{5}x\right) = 19$$

$$-4x - \frac{18}{5}x = 19$$

$$\left(\frac{5}{5}\right)\left(-\frac{4x}{1}\right) - \frac{18}{5}x = 19$$

$$-\frac{20}{5}x - \frac{18}{5}x = 19$$

$$-\frac{38}{5}x = 19$$

$$\left(-\frac{5}{38}\right)\left(-\frac{38}{5}x\right) = \frac{\cancel{19}^{1}}{1}\left(\frac{-5}{\cancel{38}_{2}}\right)$$

$$x = -\frac{5}{2}$$

4) Put $x = -\frac{5}{2}$ into either eqn:

$$-6x - 5y = 0$$

$$-\frac{6}{1}\left(-\frac{5}{2}\right) - 5y = 0$$

$$\frac{30}{2} - 5y = 0$$

$$15 - 5y = 0$$

$$\underset{-15}{} \qquad \underset{-15}{}$$

$$-5y = -15$$

$$\frac{-5y}{-5} = \frac{-15}{-5}$$

$$y = 3$$

Sol'n: $\boxed{\left(-\frac{5}{2}, 3\right)}$ or $\boxed{\left\{\left(-\frac{5}{2}, 3\right)\right\}}$

Check it: $\boxed{x = -\frac{5}{2}, \quad y = 3}$

$$-4x + 3y = 19$$
$$-\frac{4}{1}\left(-\frac{5}{2}\right) + 3(3) = 19$$
$$\frac{20}{2} + 9 = 19$$
$$10 + 9 = 19$$
$$19 = 19 \checkmark$$

$$-6x - 5y = 0$$
$$-\frac{6}{1}\left(-\frac{5}{2}\right) - 5(3) = 0$$
$$\frac{30}{2} - 15 = 0$$
$$15 - 15 = 0$$
$$0 = 0 \checkmark$$

Example: solve by substitution.

$$x + y = -1$$
$$3x - y = -3$$

1) solve $x + y = -1$ for x:

$$\begin{array}{rcl} x + y & = & -1 \\ -y & & -y \end{array}$$
$$x = -y - 1$$

2) Put $x = -y - 1$ into $3x - y = -3$

$$3(-y-1) - y = -3$$
$$-3y - 3 - y = -3$$
$$\begin{array}{rcl} -4y - 3 & = & -3 \\ +3 & & +3 \end{array}$$
$$-4y = 0$$
$$\frac{-4y}{-4} = \frac{0}{-4}$$
$$y = 0$$

solution: $(\ , 0)$ still need to find x:

3) Put $y = 0$ into $x + y = -1$:

$$x + 0 = -1$$
$$x = -1$$

Sol'n: $\boxed{(-1, 0)}$ Check it!

4.3: The Elimination method for Solving Linear Systems (Addition method)

Example: Solve the system by the elimination method (Addition method).

$$\begin{cases} -5x+2y=-6 \\ 10x+7y=34 \end{cases}$$

$$-5x+2y=-6 \xrightarrow{(2)} -10x+4y=-12$$
$$10x+7y=34 \longrightarrow 10x+7y=34$$

Add: $0x+11y=22$

$$11y=22$$
$$\frac{11y}{11}=\frac{22}{11}$$
$$y=2$$

Now either do elimination again to get rid of y, or put $y=2$ into either of the original eqns and solve.

Put $y=2$ into $-5x+2y=-6$

$$-5x+2(2)=-6$$
$$-5x+4=-6$$
$$\quad -4 \qquad -4$$
$$-5x=-10$$
$$\frac{-5x}{-5}=\frac{-10}{-5}$$
$$x=2$$

Sol'n: $\boxed{(2,2)}$

Check it!

Ex: Solve by the elimination/addition method.

$$11x - 5y = 2$$
$$3x - y = 1$$

$$11x - 5y = 2 \xrightarrow{(3)} 33x - 15y = 6$$
$$3x - y = 1 \xrightarrow{(-11)} -33x + 11y = -11$$

Add: $0x - 4y = -5$

$$-4y = -5$$
$$\frac{-4y}{-4} = \frac{-5}{-4}$$
$$y = \frac{5}{4}$$

Do elimination again to find x:

$$11x - 5y = 2 \longrightarrow 11x - 5y = 2$$
$$3x - y = 1 \xrightarrow{(-5)} -15x + 5y = -5$$

$$-4x + 0y = -3$$
$$-4x = -3$$
$$\frac{-4x}{-4} = \frac{-3}{-4}$$
$$x = \frac{3}{4}$$

Sol'n: $\left(\frac{3}{4}, \frac{5}{4}\right)$

Check:

$11x - 5y = 2$

$\frac{11}{1}\left(\frac{3}{4}\right) - \frac{5}{1}\left(\frac{5}{4}\right) = 2$

$\frac{33}{4} - \frac{25}{4} = 2$

$\frac{8}{4} = 2$

$2 = 2$ ✓

$3x - y = 1$

$\frac{3}{1}\left(\frac{3}{4}\right) - \frac{5}{4} = 1$

$\frac{9}{4} - \frac{5}{4} = 1$

$\frac{4}{4} = 1$

$1 = 1$ ✓

Ex.: Solve the system:

$$-x + 6y = 2$$
$$-18y + 3x = 5$$

Elimination

$$-x + 6y = 2 \xrightarrow{(3)} -3x + 18y = 6$$
$$3x - 18y = 5 \longrightarrow 3x - 18y = 5$$

Add: $0x + 0y = 11$

$0 = 11$ False!

Substitution.

$$-x + 6y = 2$$
$$-x = -6y + 2$$
$$\frac{-x}{-1} = -\frac{6y}{-1} + \frac{2}{-1}$$
$$x = 6y - 2$$

Put $x = 6y - 2$ into $3x - 18y = 5$

$$3(6y - 2) - 18y = 5$$
$$\cancel{18y} - 6 - \cancel{18y} = 5$$
$$-6 = 5 \quad \text{False}$$

$\boxed{\text{No Solution}}$ $\boxed{\text{Inconsistent system (lines are parallel)}}$

Ex: Solve.

$$2x - 10y = 6$$
$$-5x + 25y = -15$$

$$2x - 10y = 6 \xrightarrow{(5)} 10x - 50y = 30$$
$$-5x + 25y = -15 \xrightarrow{(2)} -10x + 50y = -30$$

Add: $0x + 0y = 0$

$0 = 0$ True!

$\boxed{\text{Dependent System: Infinitely many solutions}}$

(lines are the same)

When solving a system by elimination or substitution, and both variables disappear,

1) If you get a false statement (ex $0 = 11$ or $2 = 3$), the system is inconsistent and has no solution (lines are parallel)

2) If you get a true statement (ex: $0 = 0$ or $4 = 4$), the system is dependent and there are infinitely many solutions