

7.4: Addition and subtraction of rational expressions

Note Title

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Recall: $\frac{3}{5} \cdot \frac{4}{5} = \boxed{\frac{12}{25}}$ (multiplication)

$$\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \boxed{\frac{7}{5}}$$
 (Addition with same denominators)

$$\begin{aligned} \frac{3}{5} + \frac{4}{7} &= \frac{3}{5} \left(\frac{7}{7}\right) + \frac{4}{7} \left(\frac{5}{5}\right) \quad \text{LCD: } 5 \cdot 7 = 35 \quad \text{(Addition with different denominators)} \\ &= \frac{21}{35} + \frac{20}{35} = \frac{21+20}{35} = \boxed{\frac{41}{35}} \end{aligned}$$

Ex: Simplify. $\frac{1}{6x} + \frac{7}{6x} = \frac{1+7}{6x} = \frac{\cancel{6}^1}{\cancel{6x}_3} = \boxed{\frac{4}{3x}}$

Ex: $\frac{3}{x+5} - \frac{10}{x+5}$

$$= \frac{3-10}{x+5} = \boxed{\frac{-7}{x+5}} = \boxed{-\frac{7}{x+5}}$$

Ex: $\frac{4x}{x+1} + \frac{x+3}{x+1}$

$$= \frac{4x+x+3}{x+1} = \boxed{\frac{5x+3}{x+1}}$$

Ex: $\frac{4c}{c+5} + \frac{20}{c+5}$

$$= \frac{4c+20}{c+5} = \frac{4(\cancel{c+5})}{\cancel{c+5}} = \frac{4}{1} = \boxed{4}$$

Ex: $\frac{3y-5}{y+4} - \frac{5y-7}{y+4}$

$$= \frac{3y-5}{y+4} - \frac{(5y-7)}{y+4} = \frac{3y-5-(5y-7)}{y+4} = \frac{3y-5-5y+7}{y+4}$$

$$= \frac{-2y+2}{y+4}$$

$$= \frac{-2(y-1)}{y+4}$$

Both are correct

Ex: $\frac{3a}{a-4} - \frac{a+8}{a-4}$

$$= \frac{3a-(a+8)}{a-4}$$

$$= \frac{3a-a-8}{a-4}$$

$$= \frac{2a-8}{a-4} = \frac{2(\cancel{a-4})}{\cancel{a-4}}$$

$$= \frac{2}{1} = \boxed{2}$$

Ex: $\frac{d^2}{d-1} - \frac{8d-7}{d-1}$

$$= \frac{d^2-(8d-7)}{d-1} = \frac{d^2-8d+7}{d-1}$$

$$= \frac{(d-7)(\cancel{d-1})}{\cancel{d-1}}$$

$$= \frac{d-7}{1}$$

$$= \boxed{d-7}$$

d^2
 $\uparrow$
 $d \cdot d$
 $1d$
 7
 $\uparrow$
 $1 \cdot 7$
 $-7d$

Scratch
 d^2-8d+7
 $(d-7)(d-1)$
 Check
 $d^2-d-7d+7$
 $d^2-8d+7 \checkmark$

Ex:

$$\frac{2x+1}{x^2-1} + \frac{x+2}{x^2-1}$$

$$= \frac{2x+1+x+2}{x^2-1} = \frac{3x+3}{x^2-1} = \frac{3(x+1)}{(x+1)(x-1)}$$

$$= \boxed{\frac{3}{x-1}}$$

Adding/subtracting expressions with different denominators

Must find a common denominator.

Least Common Denominator (LCD):

Ex: $\frac{7x}{30} - \frac{11x}{12}$

$$= \frac{7x}{30} \left(\frac{2}{2}\right) + \frac{-11x}{12} \left(\frac{5}{5}\right)$$

$$= \frac{14x}{60} + \frac{-55x}{60} = \frac{14x-55x}{60}$$

LCD: 60

$$= \boxed{\frac{-41x}{60}} = \boxed{-\frac{41x}{60}}$$

Ex: $\frac{5y}{6x^2} - \frac{y}{4x}$

LCD: $12x^2$

$$= \frac{5y}{6x^2} \left(\frac{2}{2}\right) + \frac{-y}{4x} \left(\frac{3x}{3x}\right)$$

$$= \frac{10y}{12x^2} + \frac{-3xy}{12x^2}$$

$$= \boxed{\frac{10y-3xy}{12x^2}} = \boxed{\frac{y(10-3x)}{12x^2}}$$

Both are correct

Ex: $\frac{2}{xy^4} + \frac{3}{x^2y^2}$

LCD: x^2y^4

$$= \frac{2}{xy^4} \left(\frac{x}{x} \right) + \frac{3}{x^2y^2} \left(\frac{y^2}{y^2} \right)$$

$$= \frac{2x}{x^2y^4} + \frac{3y^2}{x^2y^4} = \boxed{\frac{2x+3y^2}{x^2y^4}}$$

Ex: $\frac{3}{x-3} - \frac{6}{x+5}$

LCD: $(x-3)(x+5)$

$$= \frac{3}{(x-3)} \left(\frac{x+5}{x+5} \right) + \frac{-6}{x+5} \left(\frac{x-3}{x-3} \right)$$

$$= \frac{3x+15}{(x-3)(x+5)} + \frac{-6x+18}{(x+5)(x-3)}$$

$$= \frac{3x+15-6x+18}{(x-3)(x+5)} = \boxed{\frac{-3x+33}{(x+5)(x-3)}} = \boxed{\frac{-3(x-11)}{(x+5)(x-3)}}$$

Ex: $\frac{3x}{3x-2} - \frac{5}{4x}$

LCD: $4x(3x-2)$

$$= \frac{3x}{3x-2} \left(\frac{4x}{4x} \right) + \frac{-5}{4x} \left(\frac{3x-2}{3x-2} \right)$$

$$= \frac{12x^2}{4x(3x-2)} + \frac{-15x+10}{4x(3x-2)} = \boxed{\frac{12x^2-15x+10}{4x(3x-2)}}$$

7.6: Rational Equations

Ex.: $\frac{x}{5} + \frac{1}{5} = \frac{4}{5}$

Solve.

By inspection: $x = 3$

Sol'n Set: $\boxed{\{3\}}$

Check:

$$\frac{x}{5} + \frac{1}{5} = \frac{4}{5}$$

$$x=3 \Rightarrow \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

$$\frac{4}{5} = \frac{4}{5} \checkmark$$

Ex.: $\frac{x}{8} = \frac{7}{8}$ Solve.

By inspection: $x = 7$

Sol'n Set: $\boxed{\{7\}}$

If 2 fractions are equal, and they have the same denominator, then their numerators must be equal.

Ex. Solve.

$$\frac{x}{5} - \frac{4x}{3} = \frac{1}{15}$$

LCD: 15

$$\frac{x}{5} \left(\frac{3}{3} \right) + \frac{-4x}{3} \left(\frac{5}{5} \right) = \frac{1}{15}$$

$$\frac{3x}{15} + \frac{-20x}{15} = \frac{1}{15}$$

$$\frac{3x - 20x}{15} = \frac{1}{15}$$

$$\frac{-17x}{15} = \frac{1}{15}$$

$$\cancel{15} \left(\frac{-17x}{\cancel{15}} \right) = \frac{1}{\cancel{15}} \cancel{15}$$

$$-17x = 1$$

$$\frac{-17x}{-17} = \frac{1}{-17}$$

$$x = -\frac{1}{17}$$

Sol'n Set: $\boxed{\left\{-\frac{1}{17}\right\}}$

Ex.: $\frac{4}{3} + \frac{2}{3}a = 2 - \frac{1}{3}a$

LCD: 15

multiply both sides by 15:

$$\cancel{15} \frac{4}{\cancel{3}} + \frac{2a}{3} \cancel{15} = 2 \cancel{15} - \frac{1}{3}a \cancel{15}$$

$$\frac{60}{3} + \frac{30a}{3} = 30 - \frac{15a}{3}$$

$$\underset{-12}{12} + 10a = 30 - \underset{-12}{3a}$$

$$10a = \underset{+3a}{18} - \underset{+3a}{3a}$$

$$13a = 18$$

$$\frac{13a}{13} = \frac{18}{13} \Rightarrow a = \frac{18}{13}$$

Sol'n
Set

$$\boxed{\left\{ \frac{18}{13} \right\}}$$

Ex.: Solve.

$$\frac{1}{3x} - \frac{1}{6} = \frac{5}{x}$$

LCD: 6x

multiply both sides by 6x:

$$\frac{1}{3x} \cancel{6x} - \frac{1}{6} \cancel{6x} = \frac{5}{x} \cancel{6x}$$

$$\frac{\cancel{6x}}{\cancel{3x}} - \frac{\cancel{6x}}{6} = \frac{30x}{x}$$

$$\underset{-2}{2} - x = \underset{-2}{30}$$

$$-x = 28$$

$$\frac{-x}{-1} = \frac{28}{-1}$$

$$x = -28$$

$$\boxed{\{-28\}}$$

Ex.: $\frac{2x}{x-3} - \frac{1}{2} = \frac{6}{x-3}$

LCD: 2(x-3)

$$\frac{2x}{x-3} \left(\frac{2}{2} \right) + \frac{-1}{2} \left(\frac{x-3}{x-3} \right) = \frac{6}{x-3} \left(\frac{2}{2} \right)$$

$$\frac{4x}{2(x-3)} + \frac{-x+3}{2(x-3)} = \frac{12}{2(x-3)}$$

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$$\frac{\cancel{2(x-3)}}{1} \cdot \frac{4x-x+3}{\cancel{2(x-3)}} = \frac{12}{\cancel{2(x-3)}} \cdot \frac{\cancel{(x-3)}}{1}$$

$$4x - x + 3 = 12$$

$$3x + 3 = 12$$

$$3x = 9$$

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

Check in the original:

$$\frac{2x}{x-3} - \frac{1}{2} = \frac{6}{x-3}$$

$$x=3 \Rightarrow \frac{2(3)}{3-3} - \frac{1}{2} = \frac{6}{3-3}$$

$$\frac{6}{0} - \frac{1}{2} = \frac{6}{0}$$

cannot divide
by 0!

this solution
does not check.

Throw out $x=3$.

No Solution

Important: When you solve an equation that has variables in the denominators, you must check your solutions in the original. If a "solution" creates a zero denominator, throw it out.