## 3.4: The Five-Number Summary and Boxplots

**Percentiles:**

The kth percentile, denoted $P_k$, of a data set is the value such that $k$ % of the data points are less than or equal to that value. The percentile rank of a score is the percent of scores equal to or below that score.

For example, a value is known as the 85th percentile if 85% of the data points are less than or equal to that score.

n = 50

**Example 1:** Here are the 50 randomly generated scores from Example 4 in Section 3.3. Estimate the 70th percentile, 80th percentile and the 90th percentile.

| | | | | | | |
|---|---|---|---|---|---|---|
| 37.48295 | 53.07996 | 54.94143 | 57.29676 | 60.95421 | 63.16013 | 66.48368 |
| 44.16628 | 53.20456 | 55.31494 | 57.37955 | 61.43636 | 63.3329 | 67.79641 |
| 47.40146 | 54.25092 | 55.90412 | 58.99277 | 61.91373 | 63.39574 | 67.85567 |
| 50.54246 | 54.41687 | 56.48669 | 59.10063 | 62.14886 | 63.61741 | 68.12883 |
| 51.77209 | 54.42467 | 56.64306 | 59.74812 | 62.52829 | 63.79043 | 68.23415 |
| 52.06366 | 54.87849 | 56.84053 | 60.00459 | 62.58302 | 63.93691 | 70.72309 |
| 53.05055 | 54.91449 | 57.00922 | 60.59386 | 63.15417 | 66.44211 | 73.3014 |
| | | | | | | 87.41814 |

90th percentile

80th percentile

70% below a value ⇒ 30% are above
30% of 50 is 0.30(50) = 15 ⇒ 15 values are above

80% below ⇒ 20% are above
20% of 50 is 10 ⇒ so 10 values are above

90% below ⇒ 10% are above
10% of 50 is 5. So top 5 are the top 10%

**Quartiles:**

Quartiles are values that divide a data set into fourths. The 25th percentile, 50th percentile, and 75th percentile are often referred to as the first quartiles, second quartile, and third quartile.

exact definitions differ

Method 1 (Tukey's Method): Used in our book:

The second quartile, $Q_2$ , is the median $M$ of the data set.

The first quartile, $Q_1$, is the median of the *bottom half of the data set.

The third quartile, $Q_3$, is the median of the *top half of the data set.

* If the data set has odd number of data points, the median is included in both halves.

Method 2 (NOT Used in our book):

The second quartile, $Q_2$ , is the median $M$ of the data set.

The first quartile, $Q_1$, is the median of the bottom half of the data set (the values less than $M$).

The third quartile, $Q_3$, is the median of the top half of the data set (the values greater than $M$).

**Example 2:** Calculate the quartiles for the data set $A = \{17,1,9,3,4,10,12,11,5,9,12,8,13,2,7\}$.

1 2 3 4 5 7 8 9 9 10 11 12 12 13 17

bottom half

top half

M = Q2

Bottom half: 1 2 3 4 5 7 8 9

$Q_1 = \frac{4+5}{2} = 4.5$

Top half: 9 9 10 11 12 12 13 17

$Q_3 = \frac{11+12}{2} = 11.5$

$\boxed{Q_1 = 4.5,\ Q_2 = 9,\ Q_3 = 11.5}$

**Example 3:** Calculate the quartiles for $B = \{2,3,3,4,5,5,6,7,9,9,9,10,11,12,12,13\}$.

$M = Q_2 = \frac{7+9}{2} = 8$

Bottom half: 2, 3, 3, 4, 5, 5, 6, 7

$Q_1 = \frac{4+5}{2} = 4.5$

Top half: 9, 9, 9, 10, 11, 12, 12, 13

$Q_3 = \frac{10+11}{2} = 10.5$

$Q_1 = 4.5$

$Q_2 = 8$

$Q_3 = 10.5$

**Example 4:** Calculate the quartiles for $C = \{1,2,3,8,11,15,16,19,27,29,31,34,40,51,52,52,53\}$.

**Example 5:** Calculate the quartiles for $D = \{1,1,3,5,10,10,15,15,19,20,22,24,24,30,31,32,32,38\}$.

Definition: The *interquartile range*, denoted *IQR*, is the difference between the first and third quartiles.

$$IQR = Q_3 - Q_1$$

The *IQR* is the range of the middle 50% of the data set. The interquartile range is a measure of dispersion (how spread out the data are); the standard deviation, variance, and range of the data set are also measures of dispersion. The IQR is resistant to extreme values (outliers); the range and standard deviation are not resistant to extreme values.

An *outlier* is an extreme value (extremely low or extremely high, relative to other values in the data set).

One common definition for an outlier: A data point is considered an outlier (or a potential outlier) if it lies beyond these *fences*:

Lower fence (lower limit) = $Q_1 - 1.5(IQR)$

Upper fence (upper limit) = $Q_3 + 1.5(IQR)$

So, a data point $x$ is an outlier if $x < Q_1 - 1.5(IQR)$ or if $x > Q_3 + 1.5(IQR)$.

**Example 6:** Using the definition above, find any outliers in these data sets.

a. $A = \{2,5,7,10,12,14,30\}$

m = Q2

Bottom half: 2, 5, 7, 10 ⇒ $Q_1 = \frac{5+7}{2} = 6$

Top half: 10, 12, 14, 30 ⇒ $Q_3 = \frac{12+14}{2} = 13$

Quartiles:
$Q_1 = 6$
$Q_2 = 10$
$Q_3 = 13$

$IQR = Q_3 - Q_1 = 13 - 6 = 7$

$1.5(IQR) = 1.5(7) = 10.5$

Lower fence: $Q_1 - 1.5(IQR) = 6 - 1.5(7) = -4.5$

Upper fence: $Q_3 + 1.5(IQR) = 13 + 1.5(7) = 23.5$

No data below lower fence; 30 is above upper fence

30 is an outlier

b. $B = \{2,14,16,19,23,24,30\}$

bottom half
$Q_1 = 15$

m = 19

top half
$Q_3 = 23.5$

$IQR = Q_3 - Q_1 = 23.5 - 15 = 8.5$

Lower fence: $Q_1 - 1.5(IQR) = 15 - 1.5(8.5) = 2.25$

Upper fence: $Q_3 + 1.5(IQR) = 23.5 + 1.5(8.5) = 36.25$

2 is an outlier

**Example 7:** Does the randomly generated data set in Example 1 contain any outliers?

Skip

Some researchers and statisticians consider a data point to be an extreme outlier if it lies beyond the two outer fences $Q_1 - 3(IQR)$ and $Q_3 + 3(IQR)$. Does the Example 1 data set contain extreme outliers?

**The five-number summary:**

We can get a fairly useful and descriptive picture of any data set from just 5 numbers: the minimum (smallest value), first quartile, second quartile (median), third quartile, and maximum (largest value).

| Five-number summary: | | | | |
|---|---|---|---|---|
| Minimum | $Q_1$ | $Q_2$ | $Q_3$ | Maximum |

**Boxplots:**

A boxplot, or box-and-whisker plot, visually depicts these five numbers.

How to make a boxplot:

1. Determine the minimum, quartiles, and maximum of the data set.

2. Set up a horizontal scale, and draw a box that has $Q_1$ and $Q_3$ for endpoints, and a vertical line at $Q_2$ (the median). The length of the box is $IQR = Q_3 - Q_1$.

3. Calculate the upper and lower fences, and mark them on the graph: (in pencil .... you'll erase later)

   Lower fence = $Q_1 - 1.5(IQR)$
   Upper fence = $Q_3 + 1.5(IQR)$

4. Draw a line from $Q_1$ to the smallest data point that is larger than the lower fence. Draw a line from $Q_3$ to the largest data point that is smaller than the upper fence.

5. Use an asterisk to mark any data values that lie outside the fences.

**<u>Example 8:</u>** Construct a box plot for the data set.

3, 4, 4, 5, 5, 5, 6, 6, 7, 7, 7, 7, 8, 8, 9, 11

**<u>Example 9:</u>** Construct a box plot for the data set.

21, 1, 5, 3, 7, 14, 12, 10, 5, 9, 12, 4, 6, 13, 2, 8

## Let's do a boxplot for Example 6b

2, 14, 16, 19, 23, 24, 30

From earlier:

$Q_1 = 15$

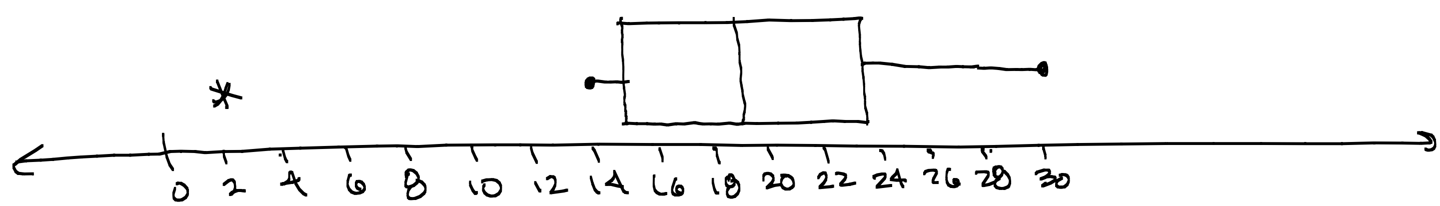
$Q_2 = 19$

$Q_3 = 23.5$

Upper fence: 36.25

lower fence: 2.25

Outlier: 2




See archived notes (Fall 2016 esp.) for more examples of boxplots.