5.1: Discrete Random Variables and Probability Distributions

A *random variable* is a quantitative variable that represents the outcomes of a probability experiment. Thus, the value of a random variable depends on chance.

A *discrete random variable* is a random variable that takes on a finite or countably infinite number of values.

A *continuous random variable* is a random variable that takes on all values on an interval of the real number line (i.e., it is not countable).

A *discrete probability distribution* is a function that assigns a probability to each outcome. (So, it assigns a probability to each value of the discrete random variable). If there are a finite number of outcomes, the sum of all their probabilities must equal 1. Each probability must be between 0 and 1, inclusive. The probability distribution can be described by a table, graph, or mathematical formula.

Notation:

If *X* is a random variable, then the probability of *X* taking on the value *x* is denoted $P(X = x)$. For example, the probability of *X* taking on the value 3 is $P(X = 3)$. The probability of *X* taking on a values of at least 5 is denoted $P(X \geq 5)$.

Example 1: A probability distribution is given by the table below.

Note: sum of probabilities is $0.32 + 0.18 + \dots + 0.08 = 1$

x	12	13	14	15	16	17	18
$P(X = x)$	0.32	0.18	0.13	0.11	0.10	0.08	0.08

a) What is $P(X = 17)$?

$P(X = 17) = \boxed{0.08}$

b) What is $P(X \geq 16)$?

$P(X \geq 16) = 0.10 + 0.08 + 0.08 = \boxed{0.26}$

c) What is $P(x > 13)$?

$C: P(X > 13) = 0.13 + 0.11 + 0.10 + 0.08 + 0.08$

OR Use complement: $P(C^c) = P(X \leq 13) = 0.32 + 0.18 = 0.5$

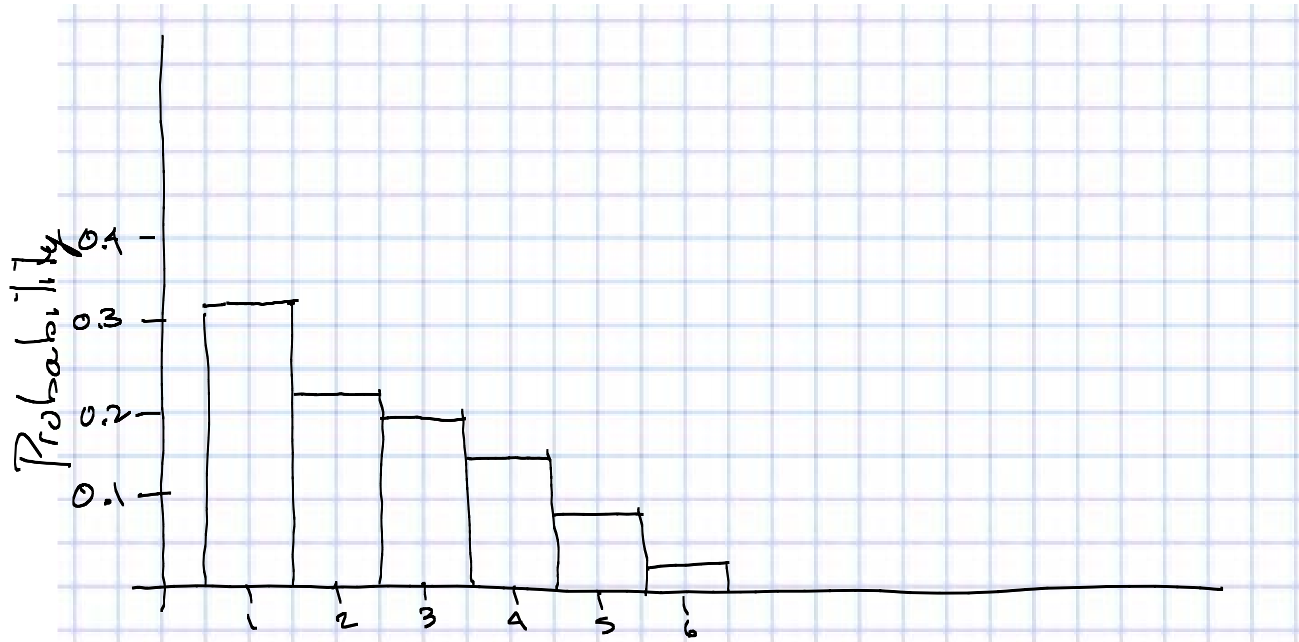
Then $P(C) = P(X > 13) = 1 - P(X \leq 13) = 1 - 0.5 = \boxed{0.5}$

Example 2: A car repair shop's records show that 25 clients have 6 cars, 83 clients have 5 cars, 140 clients have 4 cars, 183 clients have 3 cars, and 209 clients have 2 cars. The remaining 313 clients own only 1 car. Determine the probability distribution for the number of cars owned by the shop's clients. Construct a probability histogram. If the manager decides to randomly call a customer and invite him or her to complete a satisfaction survey, what is the probability that the customer called has 2 or fewer cars?

X = number of cars owned by customer

x	6	5	4	3	2	1
Freq	25	83	140	183	209	313
$P(X=x)$	25/953 ≈ 0.026	83/953 0.087	140/953 0.147	183/953 0.192	209/953 0.219	313/953 0.328

$\Sigma n = 953$



Example 3: Create a probability distribution to represent the number of girls in a three-child family. Assume that boys and girls are equally likely. Construct the probability histogram. What is the probability that a three-child family has exactly one girl? What is the probability that a three-child family has at least one girl?

b = boy, g = girl

$S = \{bbb, bbg, bgb, gbb, bgg, gbg, ggb, ggg\}$

There are 8 outcomes, all equally likely.

or $n(S) = \underbrace{2}_{1^{st}\text{ child}} \cdot \underbrace{2}_{2^{nd}\text{ child}} \cdot \underbrace{2}_{3^{rd}\text{ child}} = 8$

Let X = the number of girls

x	0	1	2	3
$P(X=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

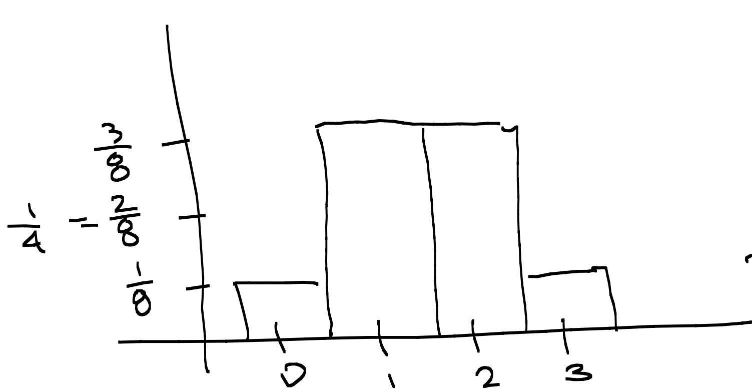
Look at the sample space to get the probabilities

Only 1 way to get 0 girls, so $P(X=0) = \frac{1}{8}$

3 ways to get 1 girl (gbb, bgb, bbg), so $P(X=1) = \frac{3}{8}$

3 ways to get 2 girls (bgg, gbg, ggb), so $P(X=2) = \frac{3}{8}$

1 way to get 3 girls (ggg), so $P(X=3) = \frac{1}{8}$



Prob. of 1 girl is $\boxed{P(X=1) = \frac{3}{8}}$

Prob. of at least 1 girl:

$P(X \geq 1) = P(X=1) + P(X=2) + P(X=3)$

$= \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \boxed{\frac{7}{8}}$