5.2: The Mean and Standard Deviation of a Discrete Random Variable

Mean of a discrete random variable:

<u>The Mean (Expected value) of a Discrete Random Variable</u>**:**

Suppose that a random variable X can take on the n values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the mean of X is

$$\mu = x_1 p_1 + x_2 p_2 + ... + x_n p_n .$$

Suppose an experiment is repeated many times, and the values of X are recorded and then averaged. As the number of repetitions increases, the average value of X will become closer and closer to μ. For that reason, the mean is called the *expected value* of X.

Example 1: A probability distribution is given by the table below. Find the mean (the expected value of X).

x	3	4	5	6	7	8	9
$P(X = x)$	0.15	0.20	0.30	0.12	0.08	0.10	0.05

$$\mu = E(X) = 3(0.15) + 4(0.20) + 5(0.3) + 6(0.12) + 7(0.08) + 8(0.10) + 9(0.05)$$

$$= \boxed{5.28}$$

Example 2: Suppose that an organization sells 1000 raffle tickets for $1 each. One ticket is for a gift basket worth $200, and three tickets are for $50 gift certificates to a restaurant. Find the expected net winnings for a person who buys one ticket.

X = Net winnings for 1 ticket

Outcome	X	$P(X)$
Win nothing	$-\$1$	$\frac{996}{1000} = 0.996$
Win gift basket	$200 - 1 = \$199$	$\frac{1}{1000} = 0.001$
Win dinner certificate	$\$50 - \$1 = \$49$	$\frac{3}{1000} = 0.003$
		Sum: 1

$$\mu = E(X) = -1(0.996) + 199(0.001) + 49(0.003) = \boxed{-\$0.65}$$

Example 3: Suppose the yearly premium for a car insurance policy is \$2300 for a customer in a certain category. Statisticians for the insurance company have determined that a person in this category has a 0.007 probability of having an accident that costs the insurance company \$100,000 and a 0.015 probability of having an accident that costs the insurance company \$30,000. What is the expected value of the insurance policy to the customer? To the insurance company?

X = value of the insurance policy to the customer

Outcome	X	$P(x)$
Big accident	$100{,}000 - 2300 = 97700$	0.007
Small accident	$30\,000 - 2300 = 27700$	0.015
No accident	-2300	$1 - 0.007 - 0.015 = 0.978$

$$E(X) = \mu = 97700(0.007) + 27700(0.015) - 2300(0.978)$$
$$= -\$1150$$

Expected value to customer is $-\$1150$.
Expected value to insurance company is \$1150.

Standard deviation of a discrete random variable:

<u>The Standard Deviation of a Discrete Random Variable</u>**:**

Suppose that a random variable X can take on the n values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the mean of X is

$$\sigma = \sqrt{(x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + ... + (x_n - \mu)^2 p_n}$$
$$= \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 p_i}$$

Variance $= \sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 p_i$

Example 4: Calculate the mean and standard deviation of the probability distribution.

x	$P(X = x)$ [sometimes written $P(x)$]
0	0.11
1	0.32
2	0.43
3	0.10
4	0.04

$\Sigma p_i = 1.00$

mean:

$$\mu = E(X) = 0(0.11) + 1(0.32) + 2(0.43) + 3(0.10) + 4(0.04) = \boxed{1.64}$$

x	$x - \mu$	$(x-\mu)^2$	$p(x)$	$(x-\mu)^2 p(x)$
0	$0 - 1.64 = -1.64$	$(-1.64)^2 = 2.6896$	0.11	$2.6896(0.11) = 0.295856$
1	$1 - 1.64 = -0.64$	$(-0.64)^2 = 0.4096$	0.32	$0.4096(0.32) = 0.131072$
2	$2 - 1.64 = 0.36$	$(0.36)^2 = 0.1296$	0.43	$0.1296(0.43) = 0.055728$
3	$3 - 1.64 = 1.36$	$(1.36)^2 = 1.8496$	0.10	$1.8496(0.10) = 0.18496$
4	$4 - 1.64 = 2.36$	$(2.36)^2 = 5.5696$	0.04	$5.5696(0.04) = 0.222784$

Variance $= \sigma^2 =$ sum $= 0.8904$

std. dev: $\sigma = \sqrt{0.8904} = \boxed{0.9436}$ (std dev)

Example 5: Use the frequencies to construct a probability distribution for the random variable X, which represents the number of games bowled by customers at a bowling alley. Calculate the mean and standard deviation of X.

X = Number of Games	Frequency	$P(X)$
1	37	37/127
2	45	45/127
3	29	29/127
4	12	12/127
5	4	4/127

n = sum = 127

Find the Mean:

$$\mu = 1\left(\frac{37}{127}\right) + 2\left(\frac{45}{127}\right) + 3\left(\frac{29}{127}\right) + 4\left(\frac{12}{127}\right) + 5\left(\frac{4}{127}\right)$$

$$= \frac{1}{127}\left(1(37) + 2(45) + 3(29) + 4(12) + 5(4)\right) = \frac{1}{127}(282) = \frac{282}{127}$$

≈ 2.2205 (store in calculator)

x	$(x-\mu)^2$	$P(x)$	$(x-\mu)^2 p(x)$
1	$(1-2.2205)^2 = 1.48955$	37/127	0.433963
2	$(2-2.2205)^2 = 0.04861$	45/127	0.017224
3	$(3-2.2205)^2 = 0.60766$	29/127	0.138757
4	$(4-2.2205)^2 = 3.16672$	12/127	0.299218
5	$(5-2.2205)^2 = 7.72577$	4/127	0.243331

Variance $= \sigma^2 =$ sum $= 1.132493$

$\sigma = \sqrt{\sigma^2}$

$= \sqrt{1.132493}$

≈ 1.064 (std. dev. of the number of games)