

9.5: Hypothesis Tests for One Population Mean When σ is Unknown

In practice, when we are using a sample to make inferences about the population mean, it is rare for us to know the population standard deviation.

Instead, we must use the sample standard deviation, s , as a point estimate of the population standard deviation, σ .

When using s as an estimate for σ , we cannot use a z -test, because $\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$ is not normally distributed.

The t -test for one population mean:

When using s as an estimate for σ , we use the Student t -distribution.

In order to use this procedure, we need to know (or be able to reasonably assume) that the variable of interest follows a normal distribution, or we must have a large sample size ($n \geq 30$).

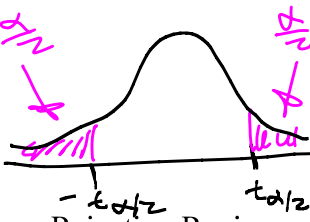
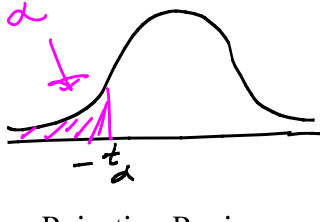
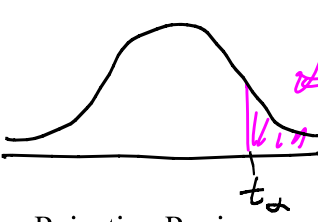
In addition, the sample should be randomly obtained, observations within the sample must be independent of one another. This means that if we have a sample size that is more than 5% of the population, we should multiply the standard error by a finite population correction factor,

$\sqrt{\frac{N-n}{n-1}}$. (In this class, I do not anticipate that we will encounter this situation.)

Hypothesis Testing for a Population Mean:

Step 1: Determine the significance level α .

Step 2: Determine the null and alternative hypotheses.

Two-Tailed Test (most common)	Left-Tailed Test (rare)	Right-Tailed Test (rare)
$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$
 Rejection Region	 Rejection Region	 Rejection Region

Note: One tailed tests assume that the scenario not listed ($\mu > \mu_0$ for a left-tailed test or $\mu < \mu_0$ for a right-tailed test) is not possible or is of zero interest.

Step 3: Use your α level and hypotheses, sketch the rejection region.

Step 4: Compute the test statistic $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$.

Step 5: Use a table (~~Table IV, on page A-13~~) to determine the critical value for t associated with your rejection region. ~~A-6, A-7~~

Step 6: Determine whether the value of t calculated from your sample (in Step 3) is in the rejection region.

- If t is in the rejection region, reject the null hypothesis.
- If t is not in the rejection region, do not reject the null hypothesis.

Step 7: State your conclusion.

Example 1: The normal human body temperature is widely accepted to be 98.6°F and can be assumed to follow a normal distribution. A medical researcher wants to know whether a certain geographical community of Native Alaskans ^{differs from} ~~has a~~ mean body temperature ~~is~~ 98.6°F . A sample of 20 members of the Native Alaskan geographical community resulted in a mean body temperature of 98.3°F with a standard deviation of 0.7°F . Perform an appropriate hypothesis test at the 95% confidence level.

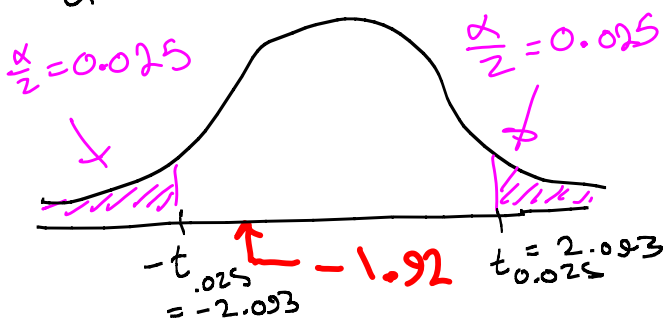
$$n=20$$

$$df=20-1=19$$

$$H_0: \mu = 98.6^{\circ}\text{F} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{2-tailed}$$

$$H_a: \mu \neq 98.6^{\circ}\text{F}$$

$$\alpha = 1 - 0.95 = 0.05$$



Find critical value of t :

From table, for $df = n - 1 = 20 - 1 = 19$ and area = 0.025 $\Rightarrow t_{0.025} = 2.093$

Example 2: The average amount of lead in the blood of young children is 2 micrograms per deciliter (mcg/dL). A city has recently changed its water supply, and there have been widespread reports of increased lead levels in the water. A concerned doctor wants to dig into the city's medical records to find out whether the children in the city have blood lead levels above the average level of 2 mcg/dL. In a sample of 35 children, she found a mean lead level of 2.60 mcg/dL with a standard deviation of 1.9 mcg/dL. Perform an appropriate hypothesis test at the 95% confidence level.

$$H_0: \mu = 2 \text{ mcg/dL}$$

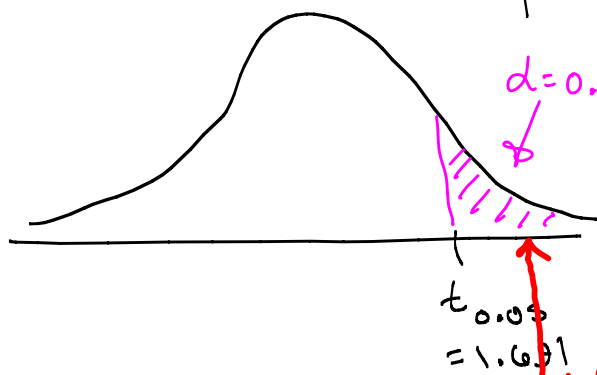
$$H_a: \mu > 2 \text{ mcg/dL}$$

$$\alpha = 1 - 0.95 = 0.05$$

Find critical value of t :

From table, for $df = 34$, area = 0.05,

$$t_{0.05} = 1.691$$



Calculate t for our sample

$$\text{std error} = \frac{s}{\sqrt{n}} = \frac{1.9}{\sqrt{35}} = 0.3216$$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{2.60 - 2}{0.3216} = 1.868$$

Calculate test statistic t

for our sample:

$$n=20$$

$$df=19$$

$$\bar{x} = 98.3^{\circ}\text{F}$$

$$s = 0.7^{\circ}\text{F}$$

$$\text{standard error} : \frac{s}{\sqrt{n}} = \frac{0.7}{\sqrt{20}} = 0.1565$$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{98.3 - 98.6}{0.1565} = -1.92$$

This t is not in the

rejection region, so we

Do Not reject H_0

This sample does not provide evidence the mean

temp differs from 98.6°F .

Sample info: $n=35$

$$df = 35 - 1 = 34$$

$$\bar{x} = 2.60$$

$$s = 1.9$$

Reject H_0

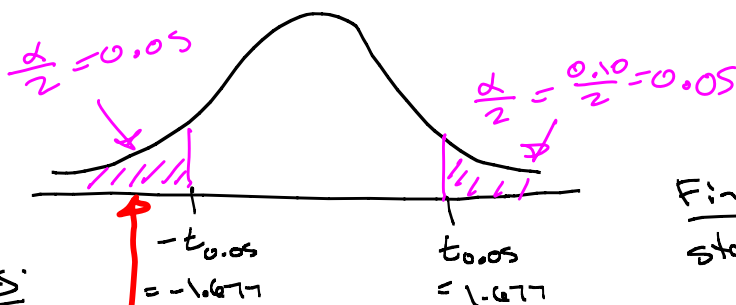
Sample provides evidence the kids' lead is above 2 mcg/dL

Example 3: Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. A customer (or a FDA analyst) buys 50 bags of chips, weighs them on a high-accuracy scale, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance.

$$H_0: \mu = 8$$

$$H_a: \mu \neq 8$$

$$\alpha = 0.10$$



Find critical values:

From table, for $df = 49$ and area $= 0.05$, critical value of t is $t_{0.05} = 1.677$

Reject H_0

This sample provides evidence the potato chip weights differ from the labeled value

Sample info $\bar{x} = 7.89$

$$s = 0.20$$

$$n = 50$$

$$df = n - 1 = 49$$

Find t for our sample

$$\text{std error: } \frac{s}{\sqrt{n}} = \frac{0.2}{\sqrt{50}} = 0.02828$$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{7.89 - 8}{0.02828} = -3.889$$

In the rejection region, so we Reject H_0

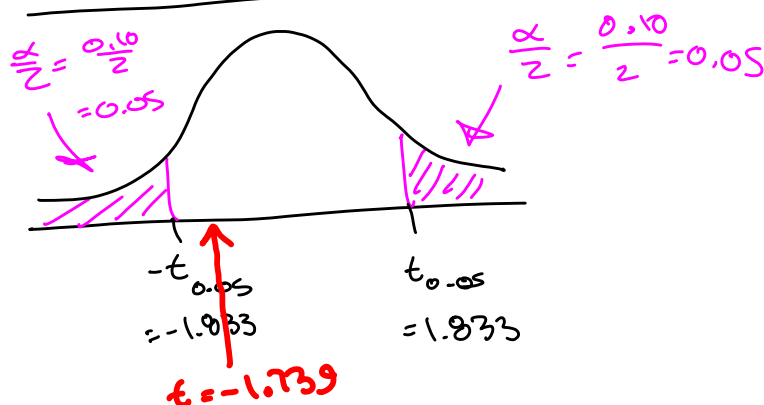
Example 4: Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. Again, a customer (or a FDA analyst) wonders whether the package size is accurate. This time, the analyst only buys 10 bags of chips, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance.

$$\alpha = 0.10$$

$$H_0: \mu = 8.02$$

$$H_a: \mu \neq 8.02$$

Find critical value at t



From table, for $df = 9$,

$$t_{0.05} = 1.833$$

Do not reject H_0

This sample does not provide evidence the mean weight differs from 8oz

Sample info:

$$n = 10$$

$$df = 10 - 1 = 9$$

$$\bar{x} = 7.89$$

$$s = 0.2$$

To do this test, we would need to assume the weights are normally distributed (since sample size < 30)

Calculate t for our sample:

$$\text{std error: } \sigma_{\bar{x}} \approx \frac{s}{\sqrt{n}} = \frac{0.2}{\sqrt{10}}$$

$$\approx 0.06325$$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{7.89 - 8}{0.06325} = -1.739$$