2.8: Linear Inequalities

Note Title 6/12/2017

$<$ means "is less than"

$>$ means "is greater than"

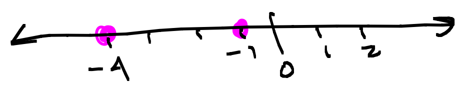
$\leq$ or $\leq$ means "is less than or equal to"

$\geq$ or $\geq$ means "is greater than or equal to

True or false?

$1 < 4$ True

$-1 < -4$ False



$5 \geq 4$ True

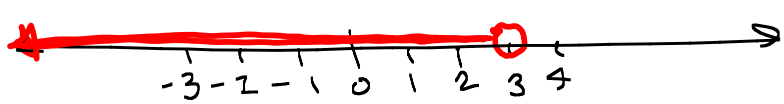
$5 \geq 5$ True

$5 > 5$ False

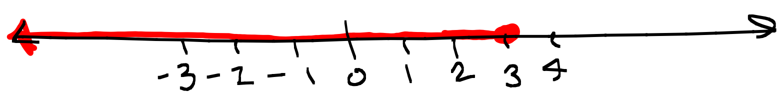
We can use number lines to represent inequalities that have variables in them.

We shade the solution set for the inequality.

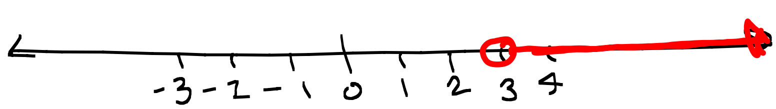
$x < 3$



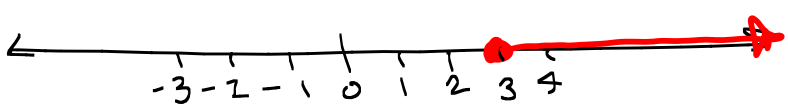
$x \leq 3$



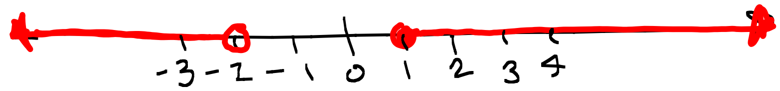
$x > 3$



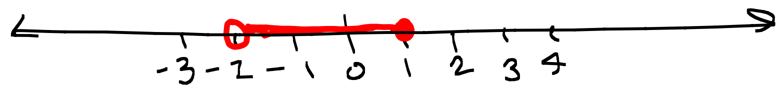
$x \geq 3$



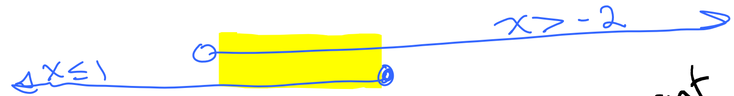
$x < -2$ or $x \geq 1$



$-2 < x \leq 1$



This means that
$-2 < x$ <u>and</u> $x \leq 1$



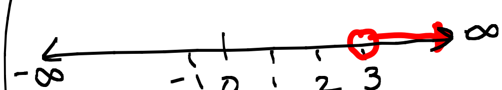
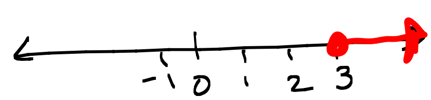
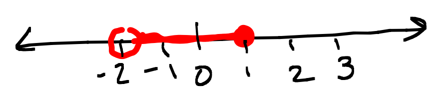
<u>Note</u>: $-2 < x$ is equivalent to $x > -2$ ← we want the overlapping part of <u>and</u> $x \geq 1$ because $x > -2$ <u>and</u>

so we have $x > -2$ <u>and</u> $x \leq 1$

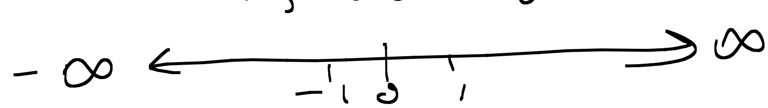
Set roster notation:

$\{x \mid x > 3\}$ "The set of all x such that $x > 3$"

↑ "such that"

Inequality	Set builder notation / Set roster notation	Interval notation	Graph
$x < 3$	$\{x \mid x < 3\}$	$(-\infty, 3)$	
$x \leq 3$	$\{x \mid x \leq 3\}$	$(-\infty, 3]$	
$x > 3$	$\{x \mid x > 3\}$	$(3, \infty)$	
$x \geq 3$	$\{x \mid x \geq 3\}$	$[3, \infty)$	
$-2 < x \leq 1$	$\{x \mid -2 < x \leq 1\}$	$(-2, 1]$	

For interval notation, think of ∞ (infinity) representing right end of number line, and $-\infty$ representing left end



Additive Property of Inequality

If $a<b$, then $a+c<b+c$

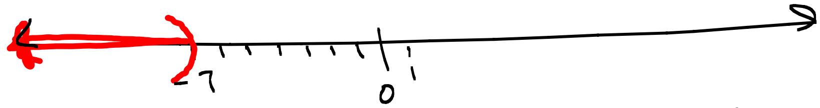
This means we can add (or subtract) the same quantity on both sides.

Example: solve $x-8<-15$

$$\begin{aligned} x-8 &< -15 \\ +8 \quad & \quad +8 \\ x &< -7 \end{aligned}$$

Sol'n set: $\boxed{\{x \mid x<-7\}}$ (set roster notation)

$\boxed{(-\infty, -7)}$ interval notion



our book does number lines this way.

Multiplicative Property of Inequality

If $a<b$ and $c>0$, then $ac<bc$.

If $a<b$ and $c<0$, then $ac>bc$.

This means: we can multiply (or divide) both sides by the same positive number.

- we can multiply (or divide) both sides by the same negative number if we reverse the inequality sign.

Why?

$1 < 2$ True

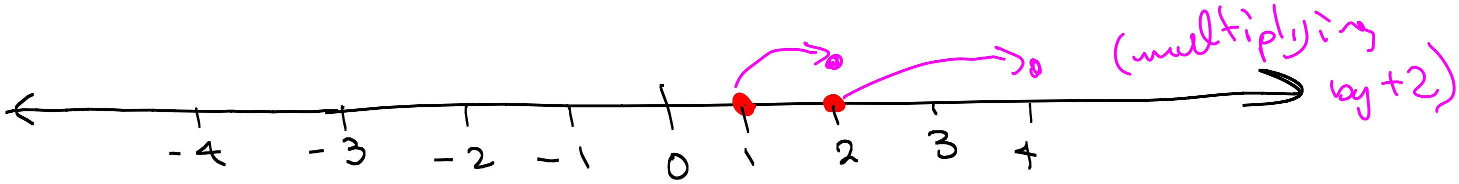
multiply both sides by 2: $1(2) < 2(2)$

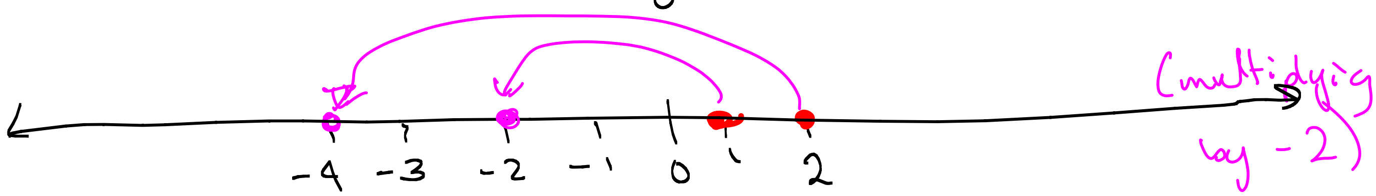
$2 < 4$ True

Start over with $1 < 2$ (still true)

multiply both sides by -2: $1(-2) < 2(-2)$ False

$-2 < -4$ False





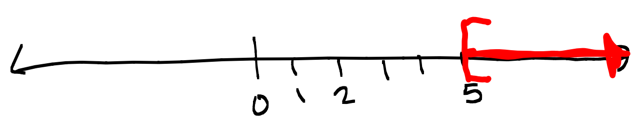
Ex: Solve.

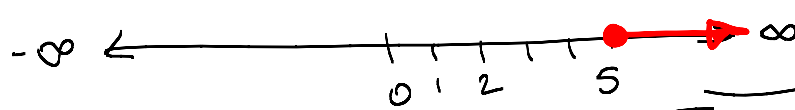
$4x \geq 20$

$\frac{4x}{4} \geq \frac{20}{4}$

$x \geq 5$

or





Interval notation: $\boxed{[5, \infty)}$

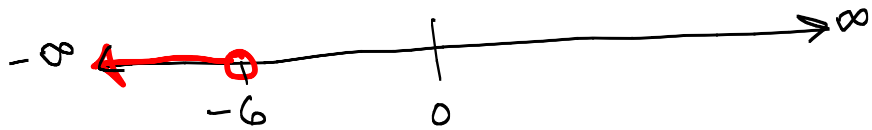
Set builder (set roster) $\boxed{\{x \mid x \geq 5\}}$

Ex:

$$-3x > 18$$

$$\frac{-3x}{-3} < \frac{18}{-3}$$ ← Reverse the inequality sign!

$$x < -6$$

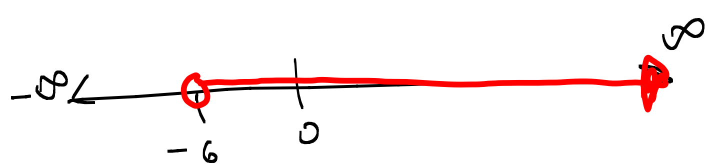


$$\boxed{(-\infty, -6)}$$

Ex: Solve $3x > -18$

$$\frac{3x}{3} > \frac{-18}{3}$$

$$x > -6$$



$$\boxed{(-6, \infty)}$$

$$\boxed{\{x \mid x > -6\}}$$

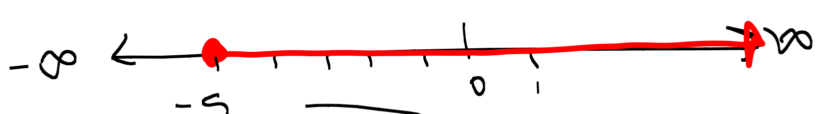
Ex: $2 \le 6y + 32$ Solve.

$\quad -32 \qquad\qquad -32$

$$-30 \le 6y$$

$$\frac{-30}{6} \le \frac{6y}{6}$$

$$-5 \le y$$



Rewrite with variable on left: $y \geq -5$

$$\boxed{\{y \mid y \geq -5\}}$$

$$\boxed{[-5, \infty)}$$

Ex: $-\frac{2}{3}x > \frac{5}{4}$

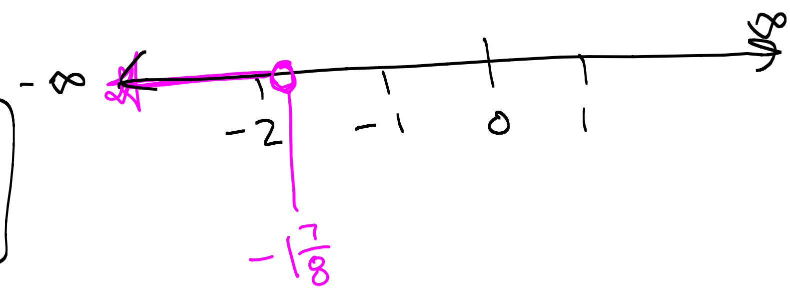
$$\frac{-\frac{2}{3}x}{-\frac{2}{3}} < \frac{\frac{5}{4}}{-\frac{2}{3}}$$

Reverse the inequality sign

$$\left(-\frac{3}{2}\right)\left(-\frac{2}{3}x\right) < \frac{5}{4}\left(-\frac{3}{2}\right)$$

$$x < -\frac{15}{8}$$

$$x < -1\frac{7}{8}$$



$$\boxed{\left\{x \mid x < -1\frac{7}{8}\right\}}$$

$$\boxed{\left(-\infty, -1\frac{7}{8}\right)}$$