

2.8: Linear Inequalities (cont'd)

Note Title

6/13/2017

When do we reverse the direction of the inequality sign?
when we divide or multiply by a negative number.

Ex: $2x - 13 \leq 5 + 8x$

$$\begin{array}{r} -8x \\ -6x - 13 \leq 5 \\ +13 +13 \end{array}$$

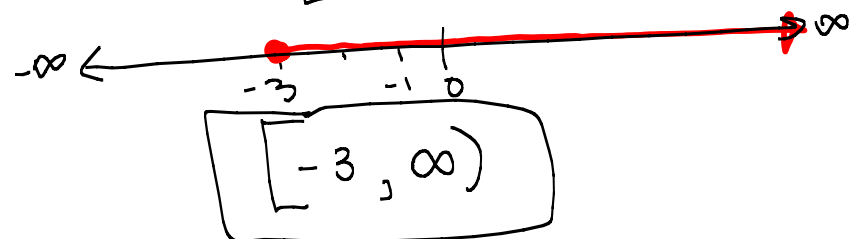
$$-6x \leq 18$$

$$\frac{-6x}{-6} \geq \frac{18}{-6}$$

$$x \geq -3$$

← change direction of inequality sign

$$\{x \mid x \geq -3\}$$



Ex: $\frac{5}{4} < -\frac{2}{3}x$

$$\frac{\frac{5}{4}}{-\frac{2}{3}} > \frac{-\frac{2}{3}x}{-\frac{2}{3}}$$

$$-\frac{3}{2} \left(\frac{5}{4} \right) > -\frac{2}{3}x \left(-\frac{3}{2} \right)$$

$$-\frac{15}{8} > x$$

Reverse the inequality sign!

See next page

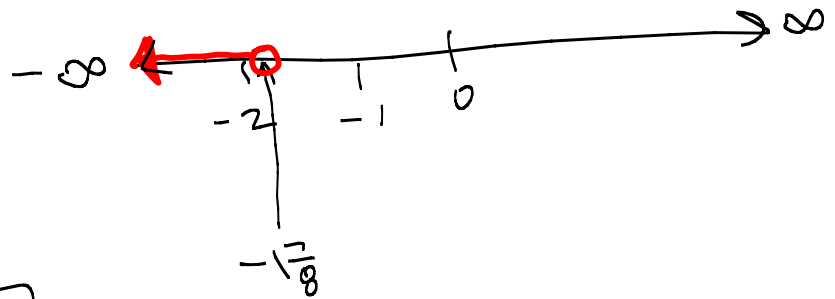
Previous example cont'd:

$$-\frac{15}{8} > x$$

Rewrite: $x < -\frac{15}{8}$

or

$$\left\{ x \mid x < -\frac{15}{8} \right\}$$
$$\left\{ x \mid x < -1\frac{7}{8} \right\}$$
$$(-\infty, -1\frac{7}{8})$$



Combined inequalities

Example: $-7 \leq 2x + 5 < 3$. Solve.

This means:

$$\begin{array}{l} -7 \leq 2x + 5 \\ -5 \quad -5 \end{array} \quad \text{and} \quad \begin{array}{l} 2x + 5 < 3 \\ -5 \quad -5 \end{array}$$

$$-12 \leq 2x$$

$$\frac{-12}{2} \leq \frac{2x}{2}$$

$$-6 \leq x$$

Rewrite: $x \geq -6$

and

and

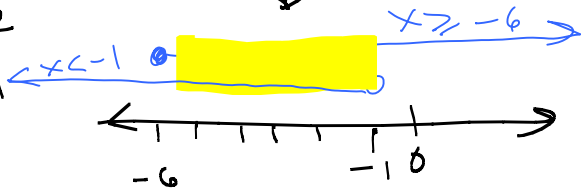
$$2x < -2$$

$$\frac{2x}{2} < \frac{-2}{2}$$

$$x < -1$$

$$x < -1$$

This section
satisfies
both



Put them back together: $-6 \leq x < -1$

Set builder: $\{x \mid -6 \leq x < -1\}$

See next
page



or

Keep them
together:

$$\begin{array}{ccc} -7 & \leq & 2x + 5 < 3 \\ -5 & & -5 \quad -5 \end{array}$$

$$-12 \leq 2x < -2$$

$$\frac{-12}{2} \leq \frac{2x}{2} < \frac{-2}{2}$$

$$-6 \leq x < -1$$

set builder
notation

$$\{x \mid -6 \leq x < -1\}$$



interval
notation

$$[-6, -1)$$

Example: $-4 < 5(3-x) \leq 1$

$$\begin{array}{c} -4 < 15 - 5x \leq 1 \\ \text{--15} \quad \text{--15} \quad \quad \quad \text{--15} \end{array}$$

$$-19 < -5x \leq -14$$

$$\frac{-19}{-5} > \frac{-5x}{-5} \geq \frac{-14}{-5}$$

Reverse the inequality signs!

$$\frac{19}{5} > x \geq \frac{14}{5}$$

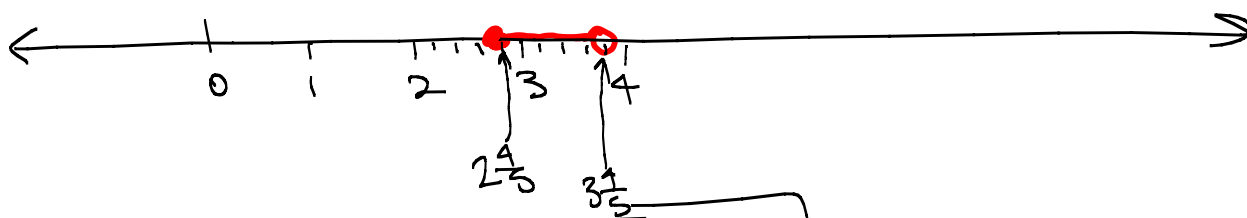
$$3\frac{4}{5} > x \geq 2\frac{4}{5}$$

Rewrite smallest to largest:

$$2\frac{4}{5} \leq x < 3\frac{4}{5}$$

$\{x \mid 2\frac{4}{5} \leq x < 3\frac{4}{5}\}$

set builder



Interval notation:

$[2\frac{4}{5}, 3\frac{4}{5})$

Ex: $-3 \leq 1+2x < -9$

$$-4 \leq 2x < -10$$

$$\frac{-4}{2} \leq \frac{2x}{2} < \frac{-10}{2}$$

$$-2 \leq x < -5$$

This means

$$-2 \leq x$$

and $x < -5$

$$x \geq -2$$

and $x < -5$



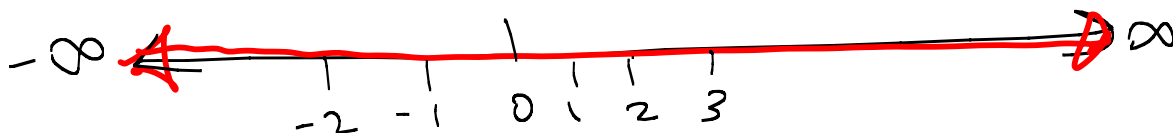
Not possible for both these statements to be true, so No Solution

How do we write

No solution using set/interval notation:

$\emptyset$
(the empty set)

How do we write All Real Numbers in interval/set notation?



$(-\infty, \infty)$