2.4 and 2.6: Applications of Linear Equations

Note Title 6/15/2017

Translating phrases into algebraic expressions:

Ex: 5 more than a number (write an algebraic expression).

Let x = the unknown number

$$x + 5$$

Ex: 3 less than twice a number

Let x = the unknown number.

$$2x - 3$$

Careful: $3 - 2x$ is incorrect

Example: A number added to 9 is the same as four times the number.

Let x represent unknown number.

$$x + 9 = 4x$$

$$-x \qquad -x$$

$$9 = 3x$$

$$\frac{9}{3} = \frac{3x}{3}$$

$$x = 3.$$

The number is 3.

Check: add to 9: $3 + 9 = 12$

4 times number: $4(3) = 12$ ✓

Example: One number is five more than twice another. If their sum is decreased by 10, the result is 22. Find the numbers.

twice another → $2x$; another → x; result is → $=$

One number: $2x+5$

another number: x

one number — compared to → another number

x

$$(2x+5)+(x)-10=22$$

(sum)

$$2x+5+x-10=22$$

$$3x-5=22$$

$+5 \quad +5$

$$3x=27$$

$$\frac{3x}{3}=\frac{27}{3}$$

$$x=9$$

another number: $x=9$

one number: $2x+5$

Substitute $x=9$: $2(9)+5$

$=18+5$

$=23$

The numbers are 9 and 23.

Check: 1st sentence:

twice 9: 18

5 more: 23 ✓

so 1st sentence checks

sum: 23

$+\ 9$

32

Decrease by 10: 22 ✓ so 2nd sentence checks

Consecutive Integer Problems

Ex: Find four consecutive integers whose sum is 74.

1st integer: x

2nd integer: $x+1$

3rd integer: $x+2$

4th integer: $x+3$

$$x + (x+1) + (x+2) + (x+3) = 74$$

$$x + x + 1 + x + 2 + x + 3 = 74$$

$$4x + 6 = 74$$

$$-6 \quad -6$$

$$4x = 68$$

$$\frac{4x}{4} = \frac{68}{4}$$

$$x = 17$$

$$4\overline{)68} \quad 17, \ 4, \ 28, \ 28, \ 0$$

1st integer: $x = 17$

2nd integer: $x+1$

Substitute $x=17$: $17+1 = 18$

3rd int: $x+2$

$x = 17 \Rightarrow 17+2 = 19$

4th int: $x+3$

$x = 17 \Rightarrow 17+3 = 20$

The integers are 17, 18, 19, 20.

Check: are they consecutive? Yes ✓

Do they add up to 74?

17
18
19
20
Sum = 74

Ex: Find three consecutive even integers whose sum is 120.

1st even integer: x

2nd even integer: $x+2$

3rd even integer: $x+4$

$$x + (x+2) + (x+4) = 120$$

$$x + x + 2 + x + 4 = 120$$

$$3x + 6 = 120$$

$$3x = 114$$

$$\frac{3x}{3} = \frac{114}{3}$$

$$x = 38$$

$$3\overline{)114}\quad 38$$

$$\begin{array}{r} 24 \\ 24 \\ \hline 0 \end{array}$$

1st integer: $x = 38$

2nd integer: $x+2$

$x = 38 \Rightarrow 38 + 2 = 40$

3rd: $x+4$

$x = 38 \Rightarrow 38 + 4 = 42$

Check it!

The integers are 38, 40, and 42.

Consecutive integers

x

$x+1$

$x+2$

$x+3$

$x+4$

Consecutive even integers

x

$x+2$

$x+4$

$x+6$

Consecutive odd integers

x

$x+2$

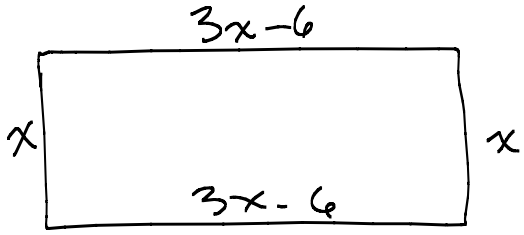
$x+4$

$x+6$

(odd integers are 2 units apart)

Ex: The perimeter of a rectangle is 100 feet. The length is 6 ft less than three times the width. Find the length and width.

length: $3x-6$
width: x



length $\xrightarrow{\text{compare to}}$ width x

Perimeter = 100

$2(\text{width})+2(\text{length})=100$ or $x+(3x-6)+x+(3x-6)=100$

$2(3x-6)+2x=100$

$6x-12+2x=100$

$8x-12=100$ → same → $8x-12=100$

$+12 \quad +12$

$8x=112$

$\frac{8x}{8}=\frac{112}{8}$

$x=14$

$8\overline{)112}$ → 14, 8, 32, 32, 0

width: $x=14$

length: $3x-6$

$x=14 \Rightarrow 3(14)-6$

$=42-6$

$=36$

The length is 36 feet and the width is 14 ft.

5.1: Multiplying and dividing expressions with common bases (Exponents Part I)

Recall: 5^3 means: $5 \cdot 5 \cdot 5 = 25 \cdot 5 = \boxed{125}$

$$-5^2 = -(5)(5) = \boxed{-25}$$

$$(-5)^2 = (-5)(-5) = \boxed{25}$$

It's a good idea to know these powers:

$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$
$2^{10} = 1024$

$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$
$3^6 = 729$

$4^2 = 16$
$4^3 = 64$
$4^4 = 256$
$4^5 = 1024$

$5^2 = 25$
$5^3 = 125$
$5^4 = 625$

$6^2 = 36$
$6^3 = 216$

$7^2 = 49$
$7^3 = 343$

$8^2 = 64$
$8^3 = 512$

$9^2 = 81$
$9^3 = 729$

$10^2 = 100$
$10^3 = 1000$
$10^4 = 10000$
$10^5 = 100000$
$10^6 = 10{,}000{,}000$

Powers of negative numbers

$$(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = \boxed{64}$$

(+) (+) (+)

$$(-2)^5 = (-2)(-2)(-2)(-2)(-2) = \boxed{-32}$$

(+) (+)

$$-2^6 = -(2)(2)(2)(2)(2)(2) = \boxed{-64}$$

$$-2^5 = -(2)(2)(2)(2)(2) = \boxed{-32}$$

not equal

equal

Important:

* If you raise a negative number to an <u>even</u> exponent, the result is <u>positive</u>.

* If you raise a negative number to an <u>odd</u> exponent, the result is <u>negative</u>.

Properties (Laws) of Exponents

5.1

1) $x^a x^b = x^{a+b}$

2) $\frac{x^a}{x^b} = x^{a-b}$

Ex: $x^2x^3 = (xx)(xxx) = x^5$

Ex: $\frac{x^6}{x^2} = \frac{xxxxxx}{xx} = \frac{xxxx}{1} = x^4$

5.2

3) $(x^a)^b = x^{ab}$

Ex: $(x^2)^3 = (xx)^3 = (xx)(xx)(xx) = x^6$

4) $(xy)^a = x^a y^a$

Ex: $(xy)^3 = (xy)(xy)(xy) = xxx\,yyy = x^3y^3$

5) $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

Ex: $\left(\frac{x}{y}\right)^3 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{x^3}{y^3}$

Ex: $3a^2(a^4) = 3a^6$

$3a^2(a^4) = 3a^2a^4 = 3a^{2+4} = 3a^6$

Ex: $\frac{6^5}{6^3} = 6^{5-3} = 6^2 = \boxed{36}$

Ex: $(3a^2)^4 = 3^4 \cdot (a^2)^4 = \boxed{81a^8}$