5.3: Negative Exponents (cont'd)

Note Title 6/20/2017

Simplify. Write variables with positive exponents only.

Example:

$$(3a^3b)\left(\frac{8ab^2}{9a^5b^3}\right)$$

$$= \frac{3a^3b}{1} \cdot \frac{8ab^2}{9a^5b^3} = \frac{\cancel{24}^{8}\, a^4 \cancel{b^3}^{1}}{\cancel{9}_{3}\, a^5 \cancel{b^3}_{1}}$$

$$= \boxed{\frac{8}{3a}} \text{ equal to } \frac{8}{3}a^{-1}$$

Ex:

$$\frac{-3x^{-5}(x^{-1}y^3z^{-2})^{-4}}{(3x^2y^4)^{-2}(2xy^{-3}z)^4} = \frac{-3x^{-5}x^4y^{-12}z^8}{(3)^{-2}x^{-4}y^{-8}(2)^4x^4y^{-12}z^4}$$

$$= \frac{-3}{x^5}\,x^4\,\frac{z^8}{y^{12}}\,(3)^2x^4y^8\,\frac{1}{(2)^4x^4}\,\frac{y^{12}}{z^4}$$

$$= \frac{-3\cdot 9\, x^8y^{20}z^8}{16x^9y^{12}z^4} = \boxed{-\frac{27y^8z^4}{16x}}$$

Ex: $5^0 = \boxed{1}$

5.4: Scientific Notation

$(57)^6 = 3.429644725 \times 10^{10}$ (from calculator)

This number is in scientific notation.

Some calculators write it as

3.429644725E10

Rewrite in non-scientific notation.

3 4 2 9 6 4 4 7 2 5 0 0 0 0

old location of decimal

10 slots

new location of decimal pt

$\boxed{34\ 296\ 447\ 250}$

or $\boxed{34{,}296{,}447{,}250.}$

Ex: write 3254 in scientific notation.

Put one non-zero digit left of the decimal pt. Then use appropriate power of 10

$3254 = \boxed{3.254 \times 10^3}$

For small numbers:

Calculator $\Rightarrow$ $\frac{1}{19^8} = 5.888045975 \times 10^{-11}$

write in regular (not scientific) notation.

$$0\ 0\ 0\ 6\ 0 . \overbrace{00\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 5}^{11 \text{ slots}}\ 888045975$$

new location of decimal

original location of decimal pt

Note: $5 \times 10^1 = 50$

$5 \times 10^2 = 500$

$5 \times 10^3 = 5000$

etc

$5 \times 10^{-1} = 5 \times \frac{1}{10} = 0.5$

$5 \times 10^{-2} = 5 \cdot \frac{1}{10^2} = 0.05$

$5 \times 10^{-3} = 5 \cdot \frac{1}{10^3} = 0.005$

$5 \times 10^{-4} = \frac{5}{10^4} = 0.0005$

$$\boxed{0.000\ 000\ 000\ 588\ 045\ 975}$$

Ex: write 0.000079148 in scientific notation

$$0.000079148 = 7.9148 \times 10^{-5}$$

5.5: Addition and Subtraction of Polynomials

A polynomial expression, or polynomial (in x), is a sum of terms of the form ax^n, where a is a real number and n is a nonnegative integer.

(only positive integers for exponents)

Examples of polynomials:

$$x^2 + 17$$

$$9x^3 - 8x^2 + 5x - 12$$

$$\frac{1}{3}x^4 - \frac{3}{7}x$$

Not polynomials:

$$x^{-5} + x^{-7} + 9x$$

$$\frac{4}{x^3} - 8 + \frac{7}{x} \quad \left(\text{could write as } 4x^{-3} - 8 + 7x^{-1}\right)$$

$$\frac{4\sqrt{x}}{\sqrt{x^5+7}}$$

Polynomials in more than 1 variable: all variables must have only positive integer exponents.

Ex: $5x^3y + 7xy + y^4$ is a polynomial in x and y.

Degree of a polynomial:

* If it has 1 variable, the degree is the largest exponent.

* If it has 2 or more variables, the degree is the largest total exponent in a term.

Ex: The degree of $-3x^4 - 7x^3 + 5x + 8$ is 4.
(this is a 4th degree polynomial.

Ex: $2x - 3x^7$ has degree 7.

Leading term: term with the highest exponent.

Leading coefficient: coefficient of the leading term.

Ex: $4x - 2x^8 + 5x^4$

Leading term: $-2x^8$

Leading coefficient: -2

Degree: 8

Often we write polynomials in order of descending powers: $-2x^8 + 5x^4 + 4x$

8th degree trinomial

Special names for certain polynomials:

Polynomial with three terms: trinomial

Polynomial with two terms: binomial

Polynomial with one term: monomial

Ex: $2 - x^3$ is a 3rd degree binomial

Ex: $5x^7$ is a 7th degree monomial

Ex: $2x^8y^2z$ is a 11th degree monomial

We add and subtract polynomials by combining like terms.

Ex: $(3x^2 - 7x + 5) + (-4x^2 + 8x + 1)$

$= 3x^2 - 7x + 5 - 4x^2 + 8x + 1$

$= \boxed{-x^2 + x + 6}$

Ex: $2x^3 - 5x^2 - 3x + 6 - (-x^2 + 5x - 8)$

$= 2x^3 - 5x^2 - 3x + 6 + x^2 - 5x + 8$

$= \boxed{2x^3 - 4x^2 - 8x + 14}$

5.6: Multiplication of Polynomials

Multiplying 2 or more monomials:

add exponents on like bases:

Ex: $(-2x^4)(3x^5) = \boxed{-6x^9}$

use property
$x^a x^b = x^{a+b}$

Ex: $(-3x^2y^3)(-4xy^6)$
$= \boxed{12x^3y^9}$

Multiplying a monomial by a polynomial with 2 or more terms.

Distribute.

Ex: $3x^3(2x^2 - 8x + 7)$

$= 3x^3(2x^2) + 3x^3(-8x) + 3x^3(7)$
$= \boxed{6x^5 - 24x^4 + 21x^3}$

Ex: $-5x^2y^3(-xy^4 + 8xy - 3x^3y^5 - 2y^2z)$

$= -5x^2y^3(-xy^4) - 5x^2y^3(8xy) - 5x^2y^3(-3x^3y^5) - 5x^2y^3(-2y^2z)$
$= 5x^3y^7 - 40x^3y^4 + 15x^5y^8 + 10x^2y^5z$

Multiplying 2 polynomials that each have 2 or more terms:

Recall: distributive property

$c(a+b) = ca + cb$ } equal

$(a+b)(c) = ac + bc$

Example: $(\underbrace{3x}_{a} + \underbrace{5}_{b})\underbrace{(x-8)}_{c}$

$= \underbrace{3x}_{a}\underbrace{(x-8)}_{c} + \underbrace{5}_{b}\underbrace{(x-8)}_{c}$ using $(a+b)(c) = ac + bc$

$= 3x^2 - 24x + 5x - 40$

$= \boxed{3x^2 - 19x - 40}$

Ex: multiply.

$(2x-1)(6x-5)$

$= 2x(6x-5) - 1(6x-5)$

$= 12x^2 - 10x - 6x + 5$

$= \boxed{12x^2 - 16x + 5}$

Ex: $(-2x+8)(x^2+7x-4)$

$$= -2x(x^2+7x-4) + 8(x^2+7x-4)$$

$$= -2x^3 - 14x^2 + 8x + 8x^2 + 56x - 32$$

$$= \boxed{-2x^3 - 6x^2 - 64x - 32}$$

Ex: $(x^2-7x-1)(2x^2-5x-6)$

$$= x^2(2x^2-5x-6) - 7x(2x^2-5x-6) - 1(2x^2-5x-6)$$

$$\begin{aligned} = 2x^4 - 5x^3 - 6x^2 & \\ -14x^3 + 35x^2 + 42x & \\ -2x^2 + 5x + 6 & \end{aligned}$$

$$= \boxed{2x^4 - 19x^3 + 27x^2 + 47x + 6}$$