

5.6: Multiplication of Polynomials (cont'd)

Note Title

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Multiply.

$$1) (4x+7)(5x-3)$$

$$2) (-x-y)(3x-2y)$$

$$3) (3y-7)(2y^3-5y^2+2y-1)$$

$$4) (3p-2)(4p+1)$$

$$5) (x-5)(x+5)$$

$$6) (x-4)^2$$

$$\begin{aligned} 1) & (4x+7)(5x-3) \\ &= 4x(5x-3) + 7(5x-3) \\ &= 20x^2 - 12x + 35x - 21 \\ &= \boxed{20x^2 + 23x - 21} \end{aligned}$$

The "FOIL" Method (for multiplying 2 binomials)

F: First

O: Outer

I: Inner

L: Last

Ex: $(2x+3)(3x-4)$

$$\begin{aligned} & 6x^2 - 8x + 9x - 12 \\ &= \boxed{6x^2 + x - 12} \end{aligned}$$

Perfect Square pattern

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

no need
to
memorize.

Ex: $(2x+1)^2$

$$= (2x+1)(2x+1)$$
$$= 4x^2 + 2x + 2x + 1$$
$$= \boxed{4x^2 + 4x + 1}$$

Ex: $(x-5)^2$

$$x^2 - 10x + 25$$
$$(x-5)(x-5)$$
$$= x^2 - 5x - 5x + 25$$
$$= \boxed{x^2 - 10x + 25}$$

Difference of 2 Squares Pattern

$$(a+b)(a-b) = a^2 - b^2$$

$(a+b)(a-b)$

$$= a^2 - \cancel{ab} + \cancel{ab} - b^2$$
$$= \boxed{a^2 - b^2}$$

Ex: $(x+4)(x-4)$

$$= x^2 - 4x + 4x - 16$$
$$= \boxed{x^2 - 16}$$

5.7: Division of Polynomials

Dividing a polynomial by a monomial

Ex. Divide.

$$\frac{6x^4 - 8x^3 + 12x^2 - 9}{2x^2}$$

Split into separate fractions:

$$\begin{aligned}\frac{6x^4 - 8x^3 + 12x^2 - 9}{2x^2} &= \frac{\overset{3}{\cancel{6}}x^4}{\underset{1}{\cancel{2}x^2}} - \frac{\overset{4}{\cancel{8}}x^3}{\underset{1}{\cancel{2}x^2}} + \frac{\overset{6}{\cancel{12}}x^2}{\underset{1}{\cancel{2}x^2}} - \frac{9}{2x^2} \\ &= \frac{3x^2}{1} - \frac{4x}{1} + \frac{6}{1} - \frac{9}{2x^2}\end{aligned}$$

$$= \boxed{3x^2 - 4x + 6 - \frac{9}{2x^2}}$$

Note:

$$\frac{2}{7} + \frac{3}{7}$$

$$= \frac{2+3}{7} = \frac{5}{7}$$

Dividing a Polynomial by a Polynomial (where divisor has 2 or more terms)

Polynomial Long Division

Ex: Divide $\frac{379}{12}$.

$$\begin{array}{r} 31 \\ 12 \overline{) 379} \\ \underline{-36} \\ 19 \\ \underline{-12} \\ 7 \end{array}$$

$$\text{So, } \frac{379}{12} = \boxed{31 + \frac{7}{12}}$$

(we usually write this as $31\frac{7}{12}$)

Terminology

$$\begin{array}{r} \text{Quotient} \\ \hline \text{Divisor} \overline{) \text{Dividend}} \\ \underline{} \\ \underline{} \\ \underline{} \\ \underline{} \\ \hline \text{Remainder} \end{array}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$$

our example: $\frac{379}{12} = 31 + \frac{7}{12}$ check: $(31)(12) + 7 = 379$

Ex: Divide.

$$\frac{3x^2 + 7x + 9}{x+2}$$

$$\begin{array}{r} 3x^2 \\ \underline{x} \\ 3x + 1 \end{array}$$

$$\begin{array}{r} x+2 \overline{) 3x^2 + 7x + 9} \\ \underline{-(3x^2 + 6x)} \\ x + 9 \\ \underline{-(x + 2)} \\ 7 \end{array}$$

Write answer:

$$\frac{3x^2 + 7x + 9}{x+2}$$

=

$$3x+1 + \frac{7}{x+2}$$

Note: could write

$$\frac{3x^2}{x+2} + \frac{7x}{x+2} + \frac{9}{x+2}$$

but they don't reduce

Need long division!

$$x(?) = 3x^2$$

Answer: $3x$

$$3x(x+2) = 3x^2 + 6x$$

$$x(?) = x$$

answer: 1

Example: Divide. $\frac{x^3 - 6x^2 + x + 14}{x - 5}$

$$\begin{array}{r}
 \frac{x^3}{x} \quad \frac{-x^2}{x} \quad \frac{-1x}{x} \\
 \downarrow \quad \downarrow \quad \downarrow \\
 x^2 - x - 4 \\
 \hline
 x - 5 \overline{) x^3 - 6x^2 + x + 14} \\
 \underline{(+)(-x^3 + 5x^2)} \\
 -x^2 + x \\
 \underline{(+)(-x^2 + 5x)} \\
 -4x + 14 \\
 \underline{(+)(-4x + 20)} \\
 -6
 \end{array}$$

write answer:

$$\frac{x^3 - 6x^2 + x + 14}{x - 5}$$

$$= x^2 - x - 4 + \frac{-6}{x - 5}$$

OR $x^2 - x - 4 - \frac{6}{x - 5}$

Check by multiplying:

$$(\text{Quotient})(\text{Divisor}) + \text{Remainder} = \text{Dividend}$$

$$\begin{aligned}
 & (x^2 - x - 4)(x - 5) - 6 \\
 = & (x - 5)(x^2 - x - 4) - 6 \\
 = & x^3 - x^2 - 4x \\
 & \quad - 5x^2 + 5x + 20 - 6 \\
 = & x^3 - 6x^2 + x + 14 \quad \checkmark
 \end{aligned}$$