5.7: Division of Polynomials (cont'd)

Note Title 6/22/2017

Ex: Divide.

$$\frac{x^5y + x^4 - 6x^3y^4}{2x^2y}$$

Break into separate fractions:

$$\frac{x^5y}{2x^2y} + \frac{x^4}{2x^2y} - \frac{6x^3y^4}{2x^2y}$$

$$= \frac{x^3}{2} + \frac{x^2}{2y} - \frac{3xy^3}{1}$$

$$= \boxed{\frac{x^3}{2} + \frac{x^2}{2y} - 3xy^3}$$

✡

Important: In long division

Arrange terms in descending order, starting with the largest exponent.

✡ Insert "placeholder" (ex $0x^3$, $0x^2$) for missing powers of x

Example: Divide.

$$\frac{3x^4 - 50x^2 - x}{x - 4}$$

$\frac{3x^4}{x}$ $\quad$ $\frac{12x^3}{x}$ $\quad$ $\frac{-2x^2}{x}$ $\quad$ $\frac{-9x}{x}$

$$3x^3 + 12x^2 - 2x - 9$$

$$x - 4 \overline{)\ 3x^4 + 0x^3 - 50x^2 - x + 0}$$

$\pm(\mp 3x^4 \pm 12x^3)$ $\quad \leftarrow 3x^3(x-4)$

$$12x^3 - 50x^2$$

$\pm(\mp 12x^3 \pm 48x^2)$ $\quad \leftarrow 12x^2(x-4)$

$$-2x^2 - x$$

$\pm(\pm 2x^2 \mp 8x)$ $\quad \leftarrow -2x(x-4)$

$$-9x + 0$$

$\mp(\pm 9x \mp 36)$ $\quad \leftarrow -9(x-4)$

$$-36$$

Write our answer

$$\frac{3x^4 - 50x^2 - x}{x - 4} = \boxed{3x^3 + 12x^2 - 2x - 9 + \frac{-36}{x-4}}$$

or $\boxed{3x^3 + 12x^2 - 2x - 9 - \frac{36}{x-4}}$

Check by multiplication:

$$(x-4)(3x^3 + 12x^2 - 2x - 9) - 36$$

$$= 3x^4 + 12x^3 - 2x^2 - 9x$$
$$- 12x^3 - 48x^2 + 8x + 36 - 36$$
$$= 3x^4 - 50x^2 - x \quad \text{OK}$$

Ex: Divide.

$$\frac{10x^3+21x^2-5}{2x^2+5x+2}$$

$\frac{10x^3}{2x^2}$ $\quad$ $\frac{-4x^2}{2x^2}$

$$2x^2+5x+2 \;\overline{)\; 10x^3+21x^2+0x-5}$$

$$5x-2$$

$$\pm(-10x^3 \mp 25x^2 \mp 10x) \quad \leftarrow 5x(2x^2+5x+2)$$

$$-4x^2-10x-5$$

$$-(\pm 4x^2 \pm 10x \pm 4) \quad \leftarrow -2(2x^2+5x+2)$$

$$-1$$

Note: in general, the degree of the remainder is 1 less than the degree of the divisor

Write answer:
$$\frac{10x^3+21x^2-5}{2x^2+5x+2} = 5x-2+\frac{-1}{2x^2+5x+2}$$

Check it:
$$(5x-2)(2x^2+5x+2)-1$$
$$= 10x^3+25x^2+10x$$
$$-4x^2-10x-4-1$$
$$= 10x^3+21x^2-5 \;\checkmark$$

Chapter 6: Factoring Polynomials

6.1: Greatest Common Factor and Factoring by Grouping

Example: $(x+3)(x+4)$ ← factored form

$= x^2 + 7x + 12$ ← unfactored form

To find the Greatest Common Factor (GCF):

* Find largest number that divides into all the coefficients.
* Find highest power of each variable that is a factor of all the terms
  (if a variable appears in all the terms, choose the smallest exponent)
* multiply these together to get the GCF.

Ex: Factor out the GCF.

GCF: $3x^2$

$15x^4 - 6x^2$

$= 3x^2\left(\frac{15x^4}{3x^2} - \frac{6x^2}{3x^2}\right)$

$= 3x^2(5x^2 - 2)$

Check: $3x^2(5x^2-2) = 15x^4 - 6x^2$ ✓

Ex: Factor.

$$48x^3 - 32x$$

GCF: $8x$

$$= 8x(6x^2 - 4)$$

Check: $8x(6x^2-4) = 48x^3 - 32x$ ✓

can still factor out 2

This means that $8x$ is not the greatest common factor

$$= 8x(2)(3x^2 - 2)$$

$$= \boxed{16x(3x^2-2)}$$

GCF: $16x$

Try again from the beginning:

$$48x^3 - 32x$$

$$= \boxed{16x(3x^2-2)}$$

check: $16x(3x^2-2) = 48x^3 - 32x$ ✓

Ex: Factor.

$$24x^3 + 18x^2 - 6$$

GCF: 6

$$= \boxed{6(4x^3 + 3x^2 - 1)}$$

check: $24x^3 + 18x^2 - 6$ ✓

Ex: $-30x^4y^3z + 12x^3y^6 - 42x^4y^2z$

GCF: $6x^3y^2$

$$= 6x^3y^2\left(\frac{-30x^4y^3z}{6x^3y^2} + \frac{12x^3y^6}{6x^3y^2} - \frac{42x^4y^2z}{6x^3y^2}\right)$$

$$= \boxed{6x^3y^2(-5xyz + 2y^4 - 7xz)}$$

Factoring by grouping:

GCF: 1

Ex: $3x^3 - 7x^2 + 15x - 35$

$= (3x^3 - 7x^2) + (15x - 35)$

Factor out GCF from each pair

$= x^2(3x-7) + 5(3x-7)$

Note: $x^2y + 5y$
$y(x^2+5)$

$= \boxed{(3x-7)(x^2+5)}$

Check: $3x^3 + 15x - 7x^2 - 35$ ✓

also correct: $\boxed{(x^2+5)(3x-7)}$

Ex: $4x^3 - 3x^2 - 36x + 27$

$(4x^3 - 3x^2) + (-36x + 27)$

$= x^2(4x-3) - 9(4x-3)$

$= \boxed{(4x-3)(x^2-9)}$

check by multiplying

$= (4x-3)(x+3)(x-3)$

6.2: Factoring Trinomials (when the leading coefficient is 1)

Ex: Factor. $x^2 + 6x + 8$ (+) same signs

$$(x + 2)(x + 4)$$

8
1 · 8
2 · 4

Check: $x^2 + 4x + 2x + 8$

$x^2 + 6x + 8$

Ex: $x^2 + 10x - 24$ (−) signs are opposite

$$(x - 2)(x + 12)$$

24
1 · 24
2 · 12
3 · 8
4 · 6

Check: $x^2 + 12x - 2x - 24$

$x^2 + 10x - 24$

signs are opposite, so I want a difference of 10

also correct: $(x+12)(x-2)$

Factor.

Ex: $x^2 - 10x + 24$ (+) signs are the same want a sum of 10

$$(x - 4)(x - 6)$$

24
1 · 24
2 · 12
3 · 8
4 · 6

Check: $x^2 - 6x - 4x + 24$

$= x^2 - 10x + 24$

Ex: $x^2 + 16x - 36$

(−) signs are opposite

want a difference of 16

36
1 · 36
2 · 18
3 · 12
4 · 9
6 · 6

$(x - 2)(x + 18)$

$x^2 + 18x - 2x - 36$

$x^2 + 16x - 36$ ✓

Ex: $-x^2 + x + 12$

(−) signs opposite

$= -1(x^2 - x - 12)$

$= -1(x + 3)(x - 4)$

12
1 · 12
2 · 6
3 · 4

Check $(x+3)(x-4)$

$= x^2 - 4x + 3x - 12$

$= x^2 - x - 12$