6.3: Factoring Trinomials (leading coefficient > 1)

Note Title 6/27/2017

Review:

Ex: Factor.

$x^2 + x - 20$ ← (−) signs are opposite, want a difference of 1x for middle term

$(x - 4)(x + 5)$

Check: $x^2 + 5x - 4x - 20$

$= x^2 + x - 20$ ✓

20
1 · 20
2 · 10
4 · 5

Ex: Factor

$x^2 - 11x + 30$ ← (+) same signs, want a sum of 11x for middle term

$(x - 5)(x - 6)$

Check: $x^2 - 6x - 5x + 30$

$x^2 - 11x + 30$

30
1 · 30
2 · 15
3 · 10
5 · 6

Ex: $2x^2 - 22x + 60$

$2(x^2 - 11x + 30)$

$2(x - 5)(x - 6)$

Ex: $-x^2 + 11x - 30$

$-1(x^2 - 11x + 30)$

$-1(x - 5)(x - 6)$

$-(x - 5)(x - 6)$

(+) signs are the same
I want a sum of 11

Ex: $x^2 - 11xy + 30y^2$

$(x - 5y)(x - 6y)$

30
1 · 30
2 · 15
3 · 10
5 · 6

Check: $x^2 - 6yx - 5yx + 30y^2$

$x^2 - 11yx + 30y^2$

$x^2 - 11xy + 30y^2$ ✓

Ex: Factor

(−) signs are opposite
I want a difference of 11

$x^2 - 11x - 30$

30
1 · 30
2 · 15
3 · 10
5 · 6

none of these have a difference of 11

Prime

This polynomial cannot be factored. We say it is **prime.**

Prime number: can only be factored as a product of itself and 1.

Examples of prime numbers: 2, 3, 5, 7, 11, 13, 17, 19

Not prime: 4, 6, 8, 9, 12, 14, 15, 16, 18, 20, 10
(composite numbers)

Prime polynomial: can only be factored (with integer coefficients) as a product of 1 (or −1) and itself.

When leading coefficient > 1:

Ex: Factor.

$$2x^2 + 9x + 10$$

(+) signs are the same ← want a sum of $9x$ for middle term

10
1
1 · 10
2 · 5

Try: $(2x + 1)(x + 10)$

Check: $2x^2 + 20x + 1x + 10$

$2x^2 + 21x + 10$ No!

Try: $(2x + 10)(x + 1)$

Check: $2x^2 + 2x + 10x + 10$

$2x^2 + 12x + 10$ No!

Try: $(2x + 2)(x + 5)$

Check: $2x^2 + 10x + 2x + 10$

$2x^2 + 12x + 10$ No!

Try $\boxed{(2x + 5)(x + 2)}$ ← correct factorization

Check: $2x^2 + 4x + 5x + 10$

$2x^2 + 9x + 10$ ✓ok

Also correct → $\boxed{(x+2)(2x+5)}$

Ex: Factor.

(+) same signs want sum of 25x for middle term

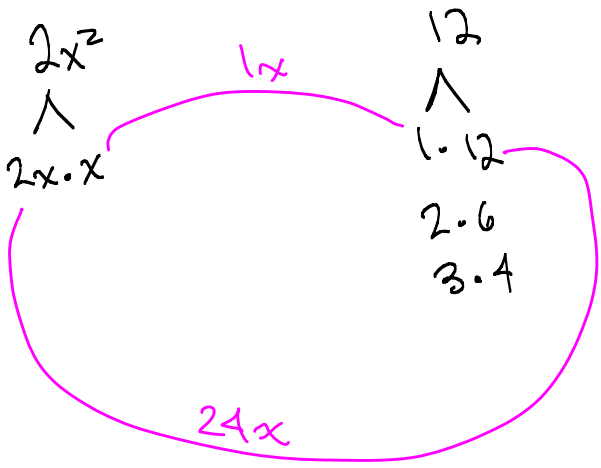
$2x^2 - 25x + 12$

$(2x - 1)(x - 12)$

Check: $2x^2 - 24x - 1x + 12$

$2x^2 - 25x + 12$

$\boxed{(2x-1)(x-12)}$



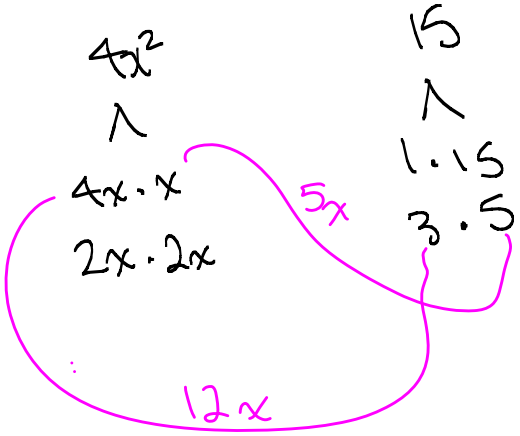
Ex: $4x^2 - 7x - 15$

(−) signs are opposite want a difference of 7x for middle term

$\boxed{(4x + 5)(x - 3)}$

Check: $4x^2 - 12x + 5x - 15$

$4x^2 - 7x - 15$ ✓



Ex: $6x^2 + 19x + 14$

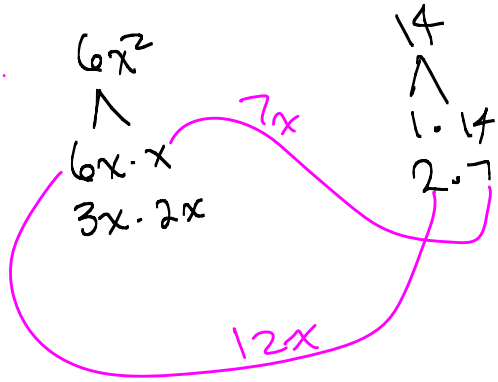
(+) same signs want a sum of 19x

$\boxed{(6x + 7)(x + 2)}$

Check: $6x^2 + 12x + 7x + 14$

$= 6x^2 + 19x + 14$ ✓

or $\boxed{(x+2)(6x+7)}$



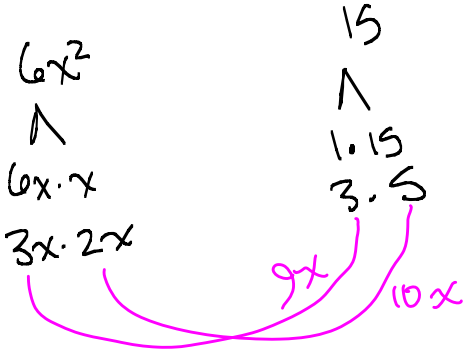
Ex: $6x^2 - x - 15$

(−) opp. signs want a difference of 1x

$\boxed{(3x - 5)(2x + 3)}$

Check: $6x^2 + 9x - 10x - 15$

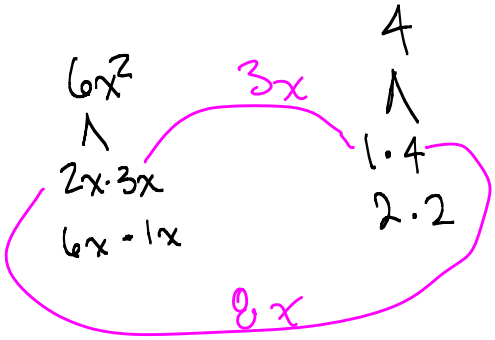
$= 6x^2 - x - 15$ ✓



Ex: $-6x^2 + 11x - 4$

$-1(6x^2 - 11x + 4)$ ← (+) same signs sum of $11x$

$\boxed{-(2x - 1)(3x - 4)}$



Check: $(2x-1)(3x-4)$

$= 6x^2 - 8x - 3x + 4$

$= 6x^2 - 11x + 4$ ✓

$-1(6x^2 - 11x + 4)$

$-6x^2 + 11x - 4$ ✓

Section 6.4: Factoring Trinomials with the AC Method

Skip the AC method

In 6.4 problems, just factor them the same way we already learned.

Section 6.5: Factoring the Difference of 2 Squares and Perfect Square Trinomials

Recall: $(a+b)(a-b)$

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$

Difference of 2 Squares Factorization

$$a^2 - b^2 = (a+b)(a-b)$$

Ex: Factor.

use $a^2 - b^2 = (a+b)(a-b)$ with $a = x$ and $b = 5$

$x^2 - 25$

$= (x)^2 - (5)^2$ (a^2, b^2)

$= (x+5)(x-5)$ ($a+b$, $a-b$)

$= \boxed{(x+5)(x-5)}$ or $\boxed{(x-5)(x+5)}$

Check: $x^2 + 5x - 5x - 25$

$= x^2 - 25$ ✓

Ex: Factor.

$4x^2 - 9$

$(2x)^2 - (3)^2$

$(2x+3)(2x-3)$

Check: $(2x+3)(2x-3)$

$= 4x^2 - 6x + 6x - 9$

$= 4x^2 - 9$ ✓

Ex: $100y^2 - 81z^2$

$(10y)^2 - (9z)^2$

$(10y + 9z)(10y - 9z)$

Check

$100y^2 - 90yz + 90yz - 81z^2$

$= 100y^2 - 81z^2$ ✓

Ex: $x^2 + 49$

Try $(x+7)(x-7)$

$x^2 - 7x + 7x - 49$

$x^2 - 49$ No!

Try $(x+7)(x+7)$

$x^2 + 7x + 7x + 49$

$x^2 + 14x + 49$

No!

This polynomial is **prime.** (does not factor)

Important fact: $a^2 + b^2$ is always prime.

$x^2 +$ positive is always prime.

Perfect Square Trinomials

Ex: $x^2 + 18x + 81$ (+) same signs sum of $18x$

$(x+9)(x+9)$

Check: $x^2 + 9x + 9x + 81$

$x^2 + 18x + 81$

$(x+9)^2$